Math 322 - Matrix Algebra SOLUTIONS

Friday, December 4 ∗∗ *Quiz 11* Name: ____________________

1. (**4 points**) Let $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right\}$ and let $W = \operatorname{Span} \mathcal{B}$. Find $\mathbf{p}$, the orthogonal projection of $\mathbf{b} = \begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix}$ onto W and also find the $\mathcal{B}$-coordinates of $\mathbf{p}$.

   We write $\mathbf{u}_1$ and $\mathbf{u}_2$ for the vectors in $\mathcal{B}$.

$$\mathbf{p} = \frac{\mathbf{u}_1 \cdot \mathbf{b}}{\mathbf{u}_1 \cdot \mathbf{u}_1}\mathbf{u}_1 + \frac{\mathbf{u}_2 \cdot \mathbf{b}}{\mathbf{u}_2 \cdot \mathbf{u}_2}\mathbf{u}_2 = \frac{6}{6}\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \frac{9}{3}\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix}$$

$$\boxed{\mathbf{p} = \begin{pmatrix} 4 \\ 4 \\ -1 \end{pmatrix}} \qquad \boxed{[\mathbf{p}]_{\mathcal{B}} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}}$$

2. (**2 points**) True/False. No justification required.

   If $W \subseteq \mathbb{R}^n$, $\mathbf{y}$ is a vector in $\mathbb{R}^n$ and $\mathbf{p} = \operatorname{proj}_W(\mathbf{y})$ is the projection of $\mathbf{y}$ onto W, then $\mathbf{y} - \mathbf{p}$ is in $W^{\perp}$. T / F

3. (a) (**3 points**) Find an orthogonal basis for the plane in $\mathbb{R}^3$ perpendicular to $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

   Apply the Gram-Schmidt process to the basis $\left\{ \mathbf{u}_1 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \right\}$ from Quiz 10.

   We take $\mathbf{v}_1 = \mathbf{u}_1$ and $\mathbf{v}_2 = \mathbf{u}_2 - \frac{\mathbf{u}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} - \frac{6}{5}\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$ $\boxed{\mathscr{B} = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3/5 \\ -6/5 \\ 1 \end{pmatrix} \right\}}$

   (b) (**1 point**) Add a third vector to the two you found in part (a) to get an orthogonal basis for $\mathbb{R}^3$.

$$\boxed{\mathscr{B} = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -3/5 \\ -6/5 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}}$$