Math 654 - Algebraic Topology
Homework 1
Fall 2016

1. Suppose that X is a topological group. If m is the multiplication of X, we can define a new operation m_* on $\pi_n(X)$ by the composition

$$S^n \xrightarrow{(\alpha,\beta)} X \times X \xrightarrow{m} X.$$

Use the Eckmann-Hilton argument to show that this operation agrees with the usual multiplication on $\pi_n(X)$.

2. A **based space** is simply a space X with a chosen basepoint $x_0 \in X$. We say that a map $f : X \longrightarrow Y$ is based (or basepoint-preserving) if $f(x_0) = y_0$, where x_0 and y_0 are the chosen basepoints of X and Y, respectively.

Let (X, x_0) and (Y, y_0) be based spaces. We define their **wedge sum** to be

$$X \vee Y = X \coprod Y / \sim$$

where $\iota_X(x_0) \sim \iota_Y(y_0)$.

   (a) Show that the wedge sum satisfies the universal property of the coproduct for based spaces. That is, if $f : (X, x_0) \longrightarrow (Z, z_0)$ and $g : (Y, y_0) \longrightarrow (Z, z_0)$ are based maps, there is a *unique* continuous map $h : (X \vee Y, \overline{x_0}) \longrightarrow (Z, z_0)$ which makes the appropriate diagram commute.

   (b) Suppose that W is another based space satisfying the universal property described above. Show that W is homeomorphic to $X \vee Y$.

3. Let X be a space. Show that the assignment $Y \mapsto X \times Y$ defines a functor **Top** $\longrightarrow$ **Top**.

4. Let **Gp** denote the category of groups and homomorphisms, and let **Comm** denote the category of commutative rings and ring homomorphisms. Show that the assignment $R \mapsto \mathrm{Gl}_n(R)$ defines a functor Gl_n : **Comm** $\longrightarrow$ **Gp**.