Math 654 - Algebraic Topology
Homework 11
Fall 2016

1. (a) Let X and Y be CW complexes. Given an n-cell e^n_α of X and a k-cell e^k_β of Y, construct a map
$$\varphi_{\alpha,\beta} : S^{n+k-1} \longrightarrow (X^n \times Y^{k-1}) \cup (X^{n-1} \times Y^k)$$
that extends to
$$\Phi_{\alpha,\beta} : D^{n+k} \cong D^n \times D^k \xrightarrow{\Phi_\alpha \times \Phi_\beta} X^n \times Y^k.$$
(Hint: It may help to use the model $S^{n+k-1} = \partial I^{n+k}$.) This construction can be used to provide $X \times Y$ with a CW structure in which cells correspond to pairs of cells in X and Y, respectively.

   (b) Starting from the minimal CW structure on S^1, describe the CW structure on the torus $T^2 = S^1 \times S^1$ resulting from the above construction.

   (c) Show that if X and Y are finite CW complexes, then $\chi(X \times Y) = \chi(X)\chi(Y)$.

2. Use the above to write down the cellular chain complex for $X = \mathbb{RP}^2 \times \mathbb{RP}^2$, and use this to compute the homology with $\mathbb{Z}$ and $\mathbf{F}_2$ coefficients.