Math 654 - Algebraic Topology Homework 2 Fall 2016

1. Let (X, \leq) be a poset.

(a) Define a category \mathscr{X} in which each element of X defines an object of \mathscr{X} and where

$$\mathscr{X}(x,y) = \begin{cases} \{*\} & x \leq y \\ \emptyset & x \not\leq y. \end{cases}$$

Show that this is a category.

- (b) If *X* and *Y* are posets and \mathscr{X} and \mathscr{Y} are the associated categories, describe functors $\mathscr{X} \longrightarrow \mathscr{Y}$.
- 2. (a) Let $X \xleftarrow{f} A \xrightarrow{g} Y$ be functions. Show that If $g : A \hookrightarrow Y$ is surjective, then $\iota_X : X \longrightarrow X \cup_A Y$ is also surjective.
 - (b) Show that you can replace "surjective" by "bijective".
 - (c) Show that you can replace "surjective" by "injective".

Remark: You should be able to give "categorical" proofs for (a) and (b), since they hold in any category. On the other hand, (c) does not hold in the category of commutative rings: the pushout of the pair of ring homomorphisms $\mathbb{Z}/2\mathbb{Z} \longleftrightarrow \mathbb{Z} \longrightarrow \mathbb{Q}$ is the trivial ring 0.

3. Find models for the sphere S^2 and torus T^2 as *simplicial* complexes (as opposed to Δ -complexes).