Math 654 - Algebraic Topology Homework 3 Fall 2016

- 1. Given the Δ -complex structure on the Klein bottle *K* described in class, compute the homology groups $H^{\Delta}_{*}(K)$.
- 2. Let X be obtained from a simplex Δ^2 by identifying the three vertices to a single point. Compute the homology groups $H^{\Delta}_*(X)$.
- 3. Build S^3 as a Δ -complex, and use this to compute $H^{\Delta}_*(S^3)$.
- 4. Consider a ball B^3 , obtained by gluing together three $\Delta^{3's}$ along the (black, vertical) edge $\{2,3\}$ as in the picture to the right. Let X be the space obtained by the gluings $e_1 \sim f_2$, $e_2 \sim f_3$, and $e_3 \sim f_1$. In the picture, e_3 and f_3 are the "back" faces. Compute H_{*}(X).

(Hint: you should find that $H_1 \cong \mathbb{Z}/3$.)

