Math 654 - Algebraic Topology
Homework 4
Fall 2016

1. Recall that the **cone** CX on a space X is defined by

$$CX = (X \times I)/(X \times 1).$$

   Using the fact that $C\Delta^n \cong \Delta^{n+1}$, convince yourself that if X is a Δ-complex, then CX inherits the structure of a Δ-complex.

   (a) Compute $H_*^\Delta(CS^1)$.

   (b) Compute $H_*^\Delta(CS^2)$.

   (c) Compute $H_*^\Delta(CT^2)$.

   What do you expect the answer to be in general?

2. (Reduced homology)

   (a) What are the reduced (simplicial) homology groups of S^1, S^2, and S^3?

   Recall that the (unreduced) suspension of X is $SX = CX \cup_X CX$.

   (b) What are the groups $\widetilde{H}_*^\Delta(ST^2)$?

   In general, how do you expect $\widetilde{H}_*^\Delta(SX)$ to be related to $\widetilde{H}_*^\Delta(X)$?

3. If X and Y are Δ-complexes with basepoints given by a choice of 0-simplex, then the wedge $X \vee Y$ inherits a Δ-complex structure.

   (a) Find the reduced homology groups $\widetilde{H}_*^\Delta(S^1 \vee S^1)$.

   (b) Find the reduced homology groups $\widetilde{H}_*^\Delta(S^1 \vee S^2)$.