Math 654 - Algebraic Topology
Homework 7
Fall 2016

1. Show that if M is a surface, then the only possible values for $\mathrm{H}_2(M)$ are 0 and $\mathbb{Z}$. Further show that if M is orientable of genus g, then $\mathrm{H}_1(M)$ can be generated by $2g$ elements, while if M is nonorientable, then $\mathrm{H}_1(M)$ can be generated by g elements.

2. $\mathbb{RP}^3$ can be built from $\mathbb{RP}^2$ by attaching a single 3-cell. If x denotes a point in the interior of the 3-cell, then $\mathbb{RP}^3 - \{x\} \simeq \mathbb{RP}^2$. Use the long exact sequence and excision to compute $\mathrm{H}_*(\mathbb{RP}^3)$ and $\mathrm{H}_*(\mathbb{RP}^3; \mathbf{F}_2)$.

3. $\mathbb{CP}^n$ can be built from $\mathbb{CP}^{n-1}$ by attaching a $2n$-cell. (Recall that $\mathbb{CP}^1 \cong S^2$.) If x denotes a point in the interior of the $2n$-cell, then $\mathbb{CP}^n - \{x\} \simeq \mathbb{CP}^{n-1}$. Use the long exact sequence and excision to compute $\mathrm{H}_*(\mathbb{CP}^n)$.