Math 654 - Algebraic Topology
Homework 9
Fall 2016

1. Let n and k be natural numbers. Show that $\mathbb{Z}/n\mathbb{Z} \otimes \mathbb{Z}/k\mathbb{Z} \cong \mathbb{Z}/\gcd(n,k)\mathbb{Z}$.

2. Show Hom is left exact. That is, if

$$0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$$

   is short exact, then

$$0 \longrightarrow \operatorname{Hom}(D, A) \longrightarrow \operatorname{Hom}(D, B) \longrightarrow \operatorname{Hom}(D, C)$$

   is exact for any D.

3. Show that $(-) \otimes D$ is right exact. That is, show that if

$$0 \longrightarrow A \xrightarrow{i} B \xrightarrow{q} C \longrightarrow 0$$

   is short exact, then

$$A \otimes D \longrightarrow B \otimes D \longrightarrow C \otimes D \longrightarrow 0$$

   is exact. (Hint: try to construct an isomorphism $\operatorname{coker}(i \otimes \operatorname{id}_D) \cong C \otimes D$.)