Math 551 - Topology I
Homework 10
Fall 2017

1. Show that S^1 is a 1-manifold.

2. Fill in the details to show that $\mathbb{RP}^n$ is an n-manifold.

3. (a) Show that if A is discrete, then $X \times A \cong \coprod_A X$.

   (b) Show directly (in other words, without using Theorem 20.2) that if X and Y are any spaces and A is discrete, then there is a bijection

   $$\mathcal{C}(X \times A, Y) \cong \mathcal{C}(X, Y^A)$$

   if Y^A is given the product topology.

   (c) Show that if A is discrete, then the compact-open topology on $\mathcal{C}(A, Y)$ agrees with the product topology on Y^A.

4. Let Y be locally compact Hausdorff and X and Z be arbitrary. Show that the "composition" map

   $$\mathrm{Map}(Y, Z) \times \mathrm{Map}(X, Y) \longrightarrow \mathrm{Map}(X, Z)$$

   defined by $(g, f) \mapsto gf$ is continuous.

5. Show that if $A \longrightarrow Y$ is an injective function (of sets), then $X \longrightarrow X \cup_A Y$ is injective as well.