Math 551 - Topology I
Homework 4
Fall 2017

1. Let $X = \mathbb{R}_{\text{cofinite}}$ be the real line equipped with the cofinite topology.
   (a) Show that if $\{x_n\}$ is a sequence *with no repeated terms*, then $\{x_n\}$ converges to **every real number**.
   (b) Show that the alternating sequence $(1, -1, 1, -1, \dots)$ does not converge.
   (c) Show that the sequence (1,2,1,3,1,4,1,5,...) converges to 1.

2. (The line with doubled origin) Let $X = \mathbb{R} \cup \{0'\}$ with topology as follows: a subset $U \subseteq \mathbb{R}$ is open if it is open in the usual topology on $\mathbb{R}$. For a subset $V \subseteq X$ that contains the new origin $0'$, we declare it to be open if $(V - \{0'\}) \cup \{0\}$ is open in $\mathbb{R}$. That is, we replace $0'$ by 0 and ask if that is open in the usual sense. Show that the sequence $(1/n)$ converges to **both** 0 and $0'$.

3. Consider the topology on $\mathbb{R}$ given by the basis consisting of open intervals (a, ∞).
   (a) Given a subset $A \subseteq \mathbb{R}$, describe the closure $\overline{A}$ in this topology.
   (b) Consider the sequence $x_n = n$. Does it converge? If so, to what?
   (c) Show that the sequence lemma holds in this topology. That is, if $x \in \overline{A}$, then some sequence in A converges to x.

   This is an example of a non-Hausdorff space in which the sequence lemma holds.

4. Consider the generic point topology on a set X, with generic point $p \in X$.
   (a) To which point(s) in X does the constant sequence $x_n = p$ converge?
   (b) Now let x_n be a sequence in X which avoids, p, that is, $x_n \neq p$ for all n. To which point(s) in X does x_n converge?

5. Let X be Hausdorff, let $A \subseteq X$, and let A' denote the set of accumulation points of A. Show that A' is closed in X.