Math 551 - Topology I
Homework 6
Fall 2017

1. Suppose that $(g, x) \mapsto g \cdot x$ is a left action of G on X. Show that the assignment $(x, g) \mapsto g^{-1} \cdot x$ defines a right action of G on X.

2. Let X be a set. For any natural number n, let Σ_n denote the symmetric group on n letters.

   (a) Show that the assignment $(x_i) \mapsto (x_{\sigma(i)})$ defines a right action of Σ_n on X^n. (Hint: One way to think about X^n is as the set of functions $\mathbf{n} \longrightarrow X$, where $\mathbf{n} = \{1, \ldots, n\}$.)

   (b) Describe the quotient $X^n \longrightarrow X^n/\Sigma_n$

   (c) Let $C_n \leq \Sigma_n$ be the cyclic subgroup of size n generated by the n-cycle $(1\,2 \cdots n)$. Describe the quotient $X^n \longrightarrow X^n/C_n$.

3. Let G be a topological group and $H \leq G$ a subgroup. Show that the closure $\overline{H} \subseteq G$ is a subgroup.

4. Let G be a topological group acting on the space X. Show that the quotient map $X \longrightarrow X/G$ is an open map.

5. (a) Show that the action of $Gl_n(\mathbb{R})$ on $\mathbb{R}^n$ discussed in class restricts to an action of the orthogonal group $O(n)$ on S^{n-1}.

   (b) Show that the orbit space of $O(n)$ acting on $\mathbb{R}^n$ is $[0, \infty)$, equipped with its standard metric topology.