Math 551 - Topology I
Supplemental Problems
Fall 2017

1. In this problem, you will give a CW structure for $S^1 \times S^2$.

   (a) Show that $\partial I^3 = (\partial I^1 \times I^2) \cup (I^1 \times \partial I^2)$ in I^3.

   (b) Consider S^1 and S^2 with their minimal CW structures, each having two cells. Build a 3-dimensional CW complex X as follows. Start with a single 0-cell, and attach both a 1-cell and a 2-cell to this point. The result is $S^1 \vee S^2$. Now attach a 3-cell, using (a) to help you define the attaching map.

   (c) Find a homeomorphism $X \xrightarrow{\cong} S^1 \times S^2$. (Since X is compact and $S^1 \times S^2$ is Hausdorff, it suffices to find a continuous bijection.)

2. The Möbius band is the quotient $M = I^2/\sim$, where $(0,t) \sim (1, 1-t)$. Find a CW structure on M.

3. The Klein bottle is the quotient $K = I^2/\sim$ where $(0,t) \sim (1,1-t)$ and $(x,0) \sim (x,1)$. Find a CW structure on K.

4. For spaces X and Y, let $[X, Y]$ denote the set of homotopy classes of maps $X \to Y$.

   (a) Show that if Y is contractible then $[X, Y]$ contains a single element.

   (b) Show that if X is contractible and Y is path-connected (and nonempty), then $[X, Y]$ contains a single element.

   (c) Show more generally that if X is contractible, then $[X, Y]$ is in bijective correspondence with the path-components of Y.