

Math 751 - Vector Bundles

Homework 2

Fall 2018

1. Let E and E' be rank n vector bundles over B , with transition functions $g_{U,V}$ and $g'_{U,V}$. Show that an isomorphism $f : E \cong E'$ corresponds bijectively to a collection of maps $\{f_U \mid U \rightarrow Gl_n(\mathbb{R})\}$ such that

$$f_V \circ g_{U,V} = g'_{U,V} \circ f_U.$$

2. (a) Write $PS(n)$ for the set of positive-definite symmetric $n \times n$ matrices. Show that there is a homeomorphism

$$Gl_n(\mathbb{R}) \cong O(n) \times PS(n).$$

- (b) Use part (a) and problem 1 to show directly that the structure group of real vector bundle can be reduced to $O(n)$.

3. On worksheet 4, you found that the transition functions for the canonical line bundle on $\mathbb{R}P^n$ under the standard open cover $\{U_i\}$ are given by $g_{ij}(\ell) = \frac{x_j}{x_i}$. Construct an isomorphism to a line bundle whose transition functions all live in $O(1) = \{\pm 1\}$.