Math 751 - Vector Bundles
Worksheet 12
Fall 2018

1. Show that the Thom space of τ_{S^2} is $\mathbb{CP}^2$.

2. Recall that, if the base space is compact, then one model for the Thom space of a bundle E is the one-point compactification of the total space E. Let L denote the dual to γ^1 on $\mathbb{RP}^n$. Then L is (non-canonically) isomorphic to γ^1.

   (a) Show that the total space of $(L)^{\oplus k}$ on $\mathbb{RP}^n$ is $\mathbb{RP}^{n+k} - \mathbb{RP}^{k-1}$.
   (Hint: A vector λ in the fiber of (L^k) over ℓ is a linear function $\ell \longrightarrow \mathbb{R}^k$. Show that the graph of this linear function is a line in $\mathbb{R}^{(n+1)+k}$.)

   (b) Conclude that $Th_{\mathbb{RP}^n}(L^k) \cong \mathbb{RP}^{n+k}/\mathbb{RP}^{k-1}$.

3. (For those that know about Steenrod operations ...)

   Consider the map $(\mathbb{RP}^\infty)^{\times n} \longrightarrow Gr_n(\mathbb{R}^\infty)$ classifying $\oplus_n \gamma^1$. On cohomology, this sends w_n to the nth elementary symmetric polynomial. Use this map to determine the action of the Steenrod operations Sq^k on the classes w_i. (Do this for small values of n, say up to 4 or 5).