Math 751 - Vector Bundles
Worksheet 2
Fall 2018

1. Let $E \xrightarrow{p} B$ be a vector bundle of rank n and write $\Gamma(E)$ for the set of sections of E.
   (a) Show that $\Gamma(E)$ is naturally a module over the ring $\mathbb{R}^B$ of continuous, real-valued functions on B.
   (b) Show that $\Gamma(E)$ is free of rank n over $\mathbb{R}^B$ if and only if E is a trivial bundle.

2. Recall the Grassmannian $Gr_k(\mathbb{R}^n)$ of k-planes in $\mathbb{R}^n$ and its canonical bundle $E_k(\mathbb{R}^n)$. Let $V \in Gr_k(\mathbb{R}^n)$, and define $\mathcal{U}_V \subseteq Gr_k(\mathbb{R}^n)$ to be the open subset consisting of $W \in Gr_k(\mathbb{R}^n)$ such that $W \cap V^\perp = \mathbf{0}$. Find a local trivialization of $E_k(\mathbb{R}^n)$ over $\mathcal{U}_V$.

3. Express the tangent bundle of S^n as the pullback of the canonical bundle of an appropriate Grassmannian.