

Math 751 - Vector Bundles
Worksheet 3
Fall 2018

1. Let E and E' be vector bundles over X .

- (a) Show that $E \oplus E'$ is a product in the category $\mathbf{Vect}_{\mathbb{R}}(X)$ of vector bundles over X .
- (b) Show that $E \oplus E'$ is a coproduct in the category $\mathbf{Vect}_{\mathbb{R}}(X)$ of vector bundles over X .

2. Recall from HW1 that a bundle can be specified by its **transition functions**

$$\{g_{U,V} : U \cap V \longrightarrow Gl_n(\mathbb{R})\},$$

which relate to the composition

$$(U \cap V) \times \mathbb{R}^n \xleftarrow{\varphi_U^{-1}} p^{-1}(U \cap V) \xrightarrow{\varphi_V} (U \cap V) \times \mathbb{R}^n$$

according to the formula

$$\varphi_V \varphi_U^{-1}(x, \mathbf{v}) = (x, g_{U,V}(\mathbf{v})).$$

Let E and E' be vector bundles over X of rank n and n' , with transition functions $\{g_{U,V}\}$ and $\{g'_{U,V}\}$.

- (a) Find the transition functions for $E \oplus E'$.
- (b) Find the transition functions for $E \otimes E'$.
- (c) Find the transition functions for E^* .
- (d) Find the transition functions for $\text{Hom}(E, E')$. (Hint: recall that $\text{Hom}(E, E') \cong E^* \otimes E'$.)