Math 751 - Vector Bundles
Worksheet 4
Fall 2018

1. Let γ_1^n be the canonical line bundle over $\mathbb{RP}^n$.

   (a) Let $\{U_0, U_1\}$ be the open cover of $\mathbb{RP}^1$, where U_i is the space of lines in $\mathbb{R}^2$ not contained in the hyperplane $x_i = 0$. Recall that a trivialization of γ_1^1 over U_i is given by
   $$p^{-1}(U_i) \cong U_i \times \mathbb{R}, \qquad (\ell, (v_0, v_1)) \mapsto (\ell, v_i).$$
   Find a formula for the transition function g_{01}.

   (b) Consider $\mathbb{RP}^2$ with its corresponding open cover $\{U_0, U_1, U_2\}$. Find the transition functions g_{01} and g_{12}.

2. Let E and E' be vector bundles over X. Show that a section of $\mathrm{Hom}(E, E')$ corresponds precisely to a bundle map $\varphi : E \longrightarrow E'$.

3. (The Picard group) For any space X, let $\mathrm{Pic}(X)$ denote the set of isomorphism classes of line bundles on X. Show that this forms an abelian group under tensor product, where the inverse is given by the dual bundle.