

Math 751 - Vector Bundles
Worksheet 7
Fall 2018

1. We stated in class that $KU^0(S^2) \cong \mathbb{Z}[H]/(1-H)^2$. As an abelian group, this is

$$\mathbb{Z}[H]/(1-H)^2 \cong \mathbb{Z}\{1\} \oplus \mathbb{Z}\{H\}.$$

- (a) Show that H^n is nonzero for all n
- (b) Show that $H^n = 1$ if and only if $n = 0$.

2. Compute $\widetilde{KU}^*(\mathbb{C}P^2)$. (Hint: $\mathbb{C}P^2$ is the cofiber of the Hopf map $\eta : S^3 \rightarrow S^2$.)

3. The periodicity theorem gives an answer for $KU^0(S^2 \times S^2)$. Compute this same group using the cofiber sequence $S^2 \vee S^2 \hookrightarrow S^2 \times S^2 \rightarrow S^2 \wedge S^2$.