Math 751 - Vector Bundles
Worksheet 9
Fall 2018

1. Let $n \geq 2$ and let γ_n be the canonical line bundle over $\mathbb{RP}^n$. If $q : S^n \longrightarrow \mathbb{RP}^n$ is the quotient map, then we showed earlier in the course that $q^*(\gamma_n)$ is trivial. Show, however, that it is not trivial as a C_2-equivariant bundle.

2. Recall that
$$KO_G(X) \cong \begin{cases} KO(X/G) & X \text{ is a free } G\text{-space} \\ KO(X) \otimes RO(G) & X \text{ is a trivial } G\text{-space} \end{cases}$$
Compute $KO^*_{C_2}(S^2)$ for the (free) antipodal action and also for the trivial action. (Hint: For the antipodal action, the cofiber sequence $S^1 \xrightarrow{2} S^1 \longrightarrow \mathbb{RP}^2$ may be useful.) [You may want to warm up with $KU^*_{C_2}$ first.]

3. (a) Let ρ be the regular representation for C_2. Recall that $S^\rho \cong \mathbb{CP}^1$ with the complex conjugation action. Show that there is a C_2-equivariant cofiber sequence
$$C_{2+} \wedge S^1 = S^1 \vee S^1 \longrightarrow S^1 \longrightarrow S^\rho,$$
where C_2 acts on $S^1 \vee S^1$ by permuting the factors and acts trivially on the middle S^1.

   (b) Use the long exact sequence arising from the above cofiber sequence to compute $\widetilde{KU}_{C_2}(S^\rho)$.