# Math 654 - Algebraic Topology
# Homework 1
# Fall 2019

1. Suppose that $X$ is a topological group. If $m$ is the multiplication of $X$, we can define a new operation $m_*$ on $\pi_n(X)$ by the composition

$$S^n \xrightarrow{(\alpha,\beta)} X \times X \xrightarrow{m} X.$$

Use the Eckmann-Hilton argument to show that this operation agrees with the usual multiplication on $\pi_n(X)$.

2. Let $X$ be a space. Show that the assignment $Y \mapsto X \times Y$ defines a functor $\mathbf{Top} \xrightarrow{X\times -} \mathbf{Top}$.

3. Let $\mathbf{Gp}$ denote the category of groups and homomorphisms, and let $\mathbf{Comm}$ denote the category of commutative rings and ring homomorphisms. Show that the assignment $R \mapsto \mathrm{Gl}_n(R)$ defines a functor $\mathrm{Gl}_n : \mathbf{Comm} \longrightarrow \mathbf{Gp}$.

4. Let $(X, \leq)$ be a poset.

   (a) Define a category $\mathscr{X}$ in which each element of $X$ defines an object of $\mathscr{X}$ and where

   $$\mathscr{X}(x,y) = \begin{cases} \{*\} & x \leq y \\ \emptyset & x \not\leq y. \end{cases}$$

   Show that this is a category.

   (b) If $X$ and $Y$ are posets and $\mathscr{X}$ and $\mathscr{Y}$ are the associated categories, describe functors $\mathscr{X} \longrightarrow \mathscr{Y}$.