Math 654 - Algebraic Topology
Homework 10
Fall 2019

1. In this problem, A, B, and C denote finitely generated abelian groups.

   (a) Show that $\mathrm{Tor}(A, B \oplus C) \cong \mathrm{Tor}(A, B) \oplus \mathrm{Tor}(A, C)$.

   (b) Compute $\mathrm{Tor}(\mathbb{Q}, A)$. (Hint: A is finitely-generated, so we can write it in the form ...)

2. (a) If

$$0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$$

   is a short exact of abelian groups and D is an abelian group, show that there is an exact sequence

$$0 \longrightarrow \mathrm{Tor}(A, D) \longrightarrow \mathrm{Tor}(B, D) \longrightarrow \mathrm{Tor}(C, D) \longrightarrow A \otimes D \longrightarrow B \otimes D \longrightarrow C \otimes D \longrightarrow 0.$$

   Hint: Take a resolution $F_1 \longrightarrow F_0 \longrightarrow D$ of D and consider

$$\begin{array}{ccccccccc} 0 & \longrightarrow & A \otimes F_1 & \longrightarrow & B \otimes F_1 & \longrightarrow & C \otimes F_1 & \longrightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & A \otimes F_0 & \longrightarrow & B \otimes F_0 & \longrightarrow & C \otimes F_0 & \longrightarrow & 0 \end{array}$$

   (b) Compute $\mathrm{Tor}(\mathbb{Q}/\mathbb{Z}, A)$.

3. Let $\mathbb{F}$ be a field. Give an identification of $\mathrm{H}^n(X; \mathbb{F})$ as the dual vector space of $\mathrm{H}_n(X; \mathbb{F})$. (You may assume that each $\mathrm{H}^*(X; \mathbb{Z})$ is finitely-generated.)