

**Math 654 - Algebraic Topology**  
**Homework 10**  
**Fall 2019**

1. Recall that the orientable surface  $M_g$  of genus  $g$  can be obtained by gluing together  $g$  tori to a sphere  $S^2$ . Collapsing this sphere gives a quotient map  $M_g \rightarrow \bigvee_g T^2$ . Use this and the ring structure on  $H^*(T^2)$  to determine the ring structure on  $H^*(M_g)$ .
  
2. Use the ring structure on  $H^*(\mathbb{R}P^m; \mathbb{F}_2)$  to show that any map  $\mathbb{R}P^n \rightarrow \mathbb{R}P^k$ , where  $n > k$  must induce the zero map on mod 2 cohomology in positive degrees.
  
3. Compute the cohomology rings  $H^*(K; \mathbb{Z})$  and  $H^*(K; \mathbb{F}_2)$ .

4.