

Math 654 - Algebraic Topology
Homework 3
Fall 2019

1. Recall that the **cone** CX on a space X is defined by

$$CX = (X \times I)/(X \times 1).$$

Using the fact that $C\Delta^n \cong \Delta^{n+1}$, convince yourself that if X is a Δ -complex, then CX inherits the structure of a Δ -complex.

- (a) Compute $H_*^\Delta(CS^1)$.
- (b) Compute $H_*^\Delta(CS^2)$.
- (c) Compute $H_*^\Delta(CT^2)$.

What do you expect the answer to be in general?

2. (Reduced homology)

- (a) What are the reduced (simplicial) homology groups of S^1 , S^2 , and S^3 ?
Recall that the (unreduced) suspension of X is $SX = CX \cup_X CX$.
- (b) What are the groups $\tilde{H}_*^\Delta(ST^2)$?

In general, how do you expect $\tilde{H}_*^\Delta(SX)$ to be related to $\tilde{H}_*^\Delta(X)$?

3. If X and Y are Δ -complexes with basepoints given by a choice of 0-simplex, then the wedge $X \vee Y$ inherits a Δ -complex structure.
- (a) Find the reduced homology groups $\tilde{H}_*^\Delta(S^1 \vee S^1)$.
 - (b) Find the reduced homology groups $\tilde{H}_*^\Delta(S^1 \vee S^2)$.