

# Math 654 - Algebraic Topology

## Homework 6

Fall 2019

1. Show that if  $M$  is a surface, then the only possible values for  $H_2(M)$  are 0 and  $\mathbb{Z}$ . Further show that if  $M$  is orientable of genus  $g$ , then  $H_1(M)$  can be generated by  $2g$  elements, while if  $M$  is nonorientable, then  $H_1(M)$  can be generated by  $g$  elements.
2.  $\mathbb{R}P^3$  can be built from  $\mathbb{R}P^2$  by attaching a single 3-cell. If  $x$  denotes a point in the interior of the 3-cell, then  $\mathbb{R}P^3 - \{x\} \simeq \mathbb{R}P^2$ . Use the long exact sequence and excision to compute  $H_*(\mathbb{R}P^3)$  and  $H_*(\mathbb{R}P^3; \mathbb{F}_2)$ .
3.  $\mathbb{C}P^n$  can be built from  $\mathbb{C}P^{n-1}$  by attaching a  $2n$ -cell. (Recall that  $\mathbb{C}P^1 \cong S^2$ .) If  $x$  denotes a point in the interior of the  $2n$ -cell, then  $\mathbb{C}P^n - \{x\} \simeq \mathbb{C}P^{n-1}$ . Use the long exact sequence and excision to compute  $H_*(\mathbb{C}P^n)$ .