Math 654 - Algebraic Topology
Homework 9
Fall 2019

1. (a) Show that free abelian groups are **flat**, meaning that if F is free abelian and

$$0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$$

is short exact, then so is

$$0 \longrightarrow F \otimes A \longrightarrow F \otimes B \longrightarrow F \otimes C \longrightarrow 0$$

   (b) Show that if $\alpha : M \longrightarrow N$ is a homomorphism of abelian groups and F is free abelian, then $F \otimes \ker(\alpha) \cong \ker(F \otimes \alpha)$.
   (we already know that the analogous statement for coker is true even if F is not free).

   (c) Show that if C_* is a chain complex of abelian groups and F is a free abelian group, then $F \otimes \mathrm{H}_n(C_*) \cong \mathrm{H}_n(F \otimes C_*)$.

   (d) Show that if $q : C_* \longrightarrow D_*$ is a quasi-isomorphism, then so is $F \otimes C_* \xrightarrow{F \otimes q} F \otimes D_*$.

2. Show that in item (d) above, F can be replaced by a chain complex of free abelian groups. That is, show that if $q : C_* \longrightarrow D_*$ is a quasi-isomorphism, then so is the homomorphism $F_* \otimes C_* \xrightarrow{F_* \otimes q} F_* \otimes D_*$.

3. Compute the homology of $\mathbb{RP}^2 \times K$, where K is the Klein bottle.