Math 181 - Section X1
Midterm II - Spring 2009
Review

Chapter 8: Probability

Terms: Outcome, event, sample space, Pascal's triangle, probability, conditional probability, mutually exclusive (disjoint, in the text) events, independent events, complement.

The conditional probability $P(A \mid B)$, or $P(A$ given that $B)$, is measured as follows: in the set of all situations in which the event B occurs, in which proportion does the event A also occur? In other words, this is just the ratio

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}.$$

Results:

- $P(\text{not } A) = 1 - P(A)$
- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- $P(A \text{ and } B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$
- Bayes' Rule: $P(A \mid B) = \frac{P(B|A) \cdot P(A)}{P(B)}$
- Two events A and B are independent if and only if $P(A \text{ and } B) = P(A)P(B)$. Equivalently, A and B are independent if and only if

$$P(A \mid B) = P(A) \quad \textit{and} \quad P(B \mid A) = P(B).$$

- The number of ways of choosing k items from a collection of n items, keeping track of the order, is $n \cdot (n-1) \cdot \cdots \cdot (n-k+1) = \frac{n!}{(n-k)!}$
- The number of ways of choosing k items from a collection of n items, ignoring order, is $\binom{n}{k} = \frac{n!}{k!(n-k)!}$
- The number $\binom{n}{k}$ is the one that appears in the nth row of Pascal's triangle, in the kth position (rows and positions start at 0).

Chapter 13: Fair Division

Terms: Equitable, Pareto-optimal, Proportional, Envy-free.

Division procedures:

- Adjusted winner [2 players, proportional, envy-free],
- Knaster inheritance [any number of players, proportional, not envy-free],
- Bottom-up (turn-taking) strategy [2 players],
- Divide-and-choose [2 players, proportional, envy-free],
- Lone Divider [3 players, proportional, not envy-free],
- Last Diminisher [any number of players, proportional, not envy-free],
- Selfridge-Conway Envy-Free Procedure [3 players, proportional, envy-free]

Some practice problems (from Chapter 13, with solutions in the back of the text):

- Bottom-up strategy: 13, 15
- Divide-and-choose: 21
- Lone divider: 24 (no solution in the text)
- Last diminisher: 25
- Conway-Selfridge Envy-free 29: