Math 181 - Section X1
Chapter 9
Review
Spring 2009

Chapter 9: Voting Theory

Know the following terms: preference list ballot, Condorcet Winner Condition (CWC), Condorcet paradox, monotonicity, Independence of Irrelevant Alternatives (IIA), Pareto condition

Voting Methods:

- Majority rule (2 candidates)

    - whoever gets the most 1st place votes wins

    - May's Theorem says that this is the only 2-candidate voting system that (i) treats all voters equally (ii) treats all candidates equally, and (iii) satisfies monotonicity

- Condorcet Method (3 or more candidates)

    - the Condorcet winner is the candidate that beats every other candidate in head-to-head matchups

    - Advantages: finds "best compromise" winner

    - Disadvantages: does not always produce a winner (this is called the Condorcet paradox)

- Plurality (3 or more candidates)

    - whoever gets the most 1st place votes wins (Majority rule is just Plurality for 2 candidates)

    - Advantages: easy to implement

    - Disadvantages: does not always find the "best compromise" candidate (the Condorcet winner)

- Borda count

  - If there are 6 candidates, for example, each first place vote is worth 5 points, a second place vote is worth 4 points, ..., a fifth place vote is worth 1 point, and a sixth place vote is 0 points. Whoever has the most points wins.

  - Advantages: This method is especially good when you want not just a 1st place winner but also second and third place winners

  - Disadvantages: Need to keep track of more information

- Sequential pairwise voting

  - The candidates are listed in some particular order (could be random), and you first compare the first candidate to the second. Whoever wins that matchup then faces the third candidate on the list. Whoever wins that faces the fourth candidate on the list. Keep going until you exhaust the list. Whoever wins the final matchup is the winner.

  - Advantages: This method always finds the Condorcet winner

  - Disadvantages: The result depends heavily on the initial choice of ordering of the candidates

- Hare voting

  - This proceeds in stages. At each stage remove all candidates who are not tied for most first place votes. Once a candidate is removed, the remaining candidates get moved up to fill the empty spots.

  - For instance, if candidate C is removed in the first stage and one voter has a ballot that looks like $\boxed{\text{1. C, 2. B, 3. A}}$, then that ballot will become $\boxed{\text{1. B, 2. A}}$ in the next stage.

  - This method, or something similar, is used in Australia, Ireland, and other places to elect public officials.

  - Disadvantages: Does not satisfy monotonicity

Conditions voting systems should satisfy

- Condorcet Winner Condition (CWC) - a voting system should find the Condorcet winner *if* there is one

- Monotonicity - if candidate A wins and one voter switches their vote to put candidate A even *higher* on their ballot, candidate A should still win.

- Independence of Irrelevant Alternatives (IIA) - If candidate A wins and candidate B loses and some voters change their votes, then the only way that candidate B can win is if some of the voters decide to move B ahead of A.

- Pareto condition - a voting system should never choose candidate A as the winner if all voters prefer candidate B to candidate A

Arrow's Theorem: There is no perfect 3-candidate voting system!