Math 535
Homework X

Due Fri. Apr. 24

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Problem 1. Show that if X is CG and Y is CGWH then $\mathscr{C}(X,Y)$ is WH. (Hint: show that $\Delta(\mathscr{C}(X,Y))$ is closed by writing it as an intersection of closed sets.)

Problem 2. Let X and Y be CGWH such that the usual product $X \times_u Y$ is not already CG (so the identity map $X \times_u Y \to X \times Y$ is not continuous). Show that the map

$$\Phi : \mathscr{C}_u(X \times_u Y, Z) \to \mathscr{C}_u(Y, \mathscr{C}_u(X, Z))$$

is (i) defined for any CGWH Z and (ii) *not* onto when $Z = X \times Y$.

Problem 3. Show that given maps $f, g : X \to Y$ and $h, i : Y \to Z$, if $f \simeq g$ and $h \simeq i$, then $h \circ f \simeq i \circ g$.

Problem 4. For spaces X and Y, let $[X,Y]$ denote the set of homotopy classes of maps $X \to Y$.

(i) Show that if Y is contractible then $[X,Y]$ contains a single element.

(ii) Show that if X is contractible and Y is path-connected, then $[X,Y]$ contains a single element.

(iii) Show more generally that if X is contractible, then $[X,Y]$ is in bijective correspondence with the path-components of Y.

Problem 5. Show that composition of paths satisfies the following "cancellation" property: if α and β are path in X from x_0 to x_1 and γ is a path from x_1 to x_2, then $\gamma \star \alpha \simeq_p \gamma \star \beta$ implies that $\alpha \simeq_p \beta$.