Math 535
Homework IX

Due Wed. Apr. 15

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Problem 1. Let A, B, and C be sets. Show that the function

$$\Phi : C^{B\times A} \to (C^B)^A$$

defined by $\Phi(f)(a)(b) = f(a,b)$ is a bijection.

Problem 2. Given a space X, say that $W \subseteq X$ is k-closed if for every compact Hausdorff space K and continuous $u : K \to X$, $u^{-1}(W)$ is closed in K.

(i) Show that the k-closed sets in X define a new topology on X (these are the closed sets for this topology). We write $k(X)$ for the set X equipped with this new topology.

(ii) Show that the identity map $k(X) \to X$ is continuous and that it is a homeomorphism if and only if X is compactly generated.

(iii) Show that the identity map $k(k(X)) \to k(X)$ is a homeomorphism for any X. Conclude that $k(X)$ is always compactly generated.

Problem 3. Show that if X is compactly generated and Y is arbitrary, then a function $f : X \to Y$ is continuous if and only if it is continuous, considered as a function $X \to k(Y)$.

Problem 4. Let X and Y be compactly generated.

(i) Show that the projection maps $\pi_X : k(X \times Y) \to X$ and $\pi_Y : k(X \times Y) \to Y$ are continuous.

(ii) Show that the space $k(X \times Y)$ satisfies the universal property for products of compactly generated spaces. That is, if Z is any other compactly generated space with maps $f : Z \to X$ and $g : Z \to Y$ there is a unique map $h : Z \to k(X \times Y)$ such that $\pi_X \circ h = f$ and $\pi_Y \circ h = g$.

Problem 5. Show that a compactly generated space X is weak Hausdorff if and only if the diagonal $\Delta(X)] \subseteq k(X \times X)$ is closed.

Problem 6. (i) Show that if X is weak Hausdorff, then so is $k(X)$.

(ii) Show that a subspace of a weak Hausdorff space is weak Hausdorff.

(iii) Show that if X and Y are nonempty and compactly generated, then $k(X \times Y)$ is weak Hausdorff if and only if each of X and Y is weak Hausdorff.

(iv) Show that if X is compactly generated and $q : X \to Y$ is a quotient map, then Y is also compactly generated.