HOMEWORK II
MATH 527
SPRING 2011

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Problem 1. Suppose that X is a path-connected based space, so that the forgetful map $u : \pi_n(X, x) \longrightarrow [S^n, X]$ is surjective, as shown in class. Show that $u(\alpha) = u(\beta)$ if and only if there is $\gamma \in \pi_1(X, x)$ such that

$$\beta = \gamma \cdot \alpha.$$

For $n = 1$, this establishes a bijective correspondence between $[S^1, X]$ and the conjugacy classes in $\pi_1(X, x)$.

Problem 2. A connected space is said to be **simple** if $\pi_1(X)$ acts trivially on all higher homotopy groups (including the conjugation action on $\pi_1(X)$ itself. Show that an H-space is simple.

Problem 3. Let X be the topologists' sine curve, $X = \{0\} \times [-1, 1] \cup \{(x, \sin \frac{1}{x}) \mid 0 < x \leq \pi\}$. Consider the map $S^0 \longrightarrow X$ which picks out the points $(0, -1)$ and $(\pi, 0)$. Show that this map is a weak homotopy equivalence but not a homotopy equivalence.

Problem 4. Let $A = \{0\} \cup \{1/n\}_{n \in \mathbb{N}}$ and $X = I$. Show that the inclusion $A \hookrightarrow X$ is not a cofibration.

Problem 5. Give an example of a space such that *no choice* of basepoint will make it well-pointed.

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