Math 651 - Topology II
Homework III
Spring 2014

1. Let X be path-connected. Show that $\pi_1(X)$ is abelian if and only if the change-of-basepoint homomorphisms $\Phi_\alpha : \pi_1(X, x_0) \longrightarrow \pi_1(X, x_1)$ do not depend on the choice of path α.

2. Does the Borsuk-Ulam theorem also hold for the torus? That is, given a map $f : S^1 \times S^1 \longrightarrow \mathbb{R}^2$, must there be a point (x, y) such that $f(x, y) = f(-x, -y)$.

3. (a) Let X be a finite CW complex (meaning finitely many cells), and let x_0 be a point in the 0-skeleton. Modify the proof that S^n is simply-connected for $n \geq 2$ to show that the inclusion of the 1-skeleton $X^1 \hookrightarrow X$ induces a surjection

$$\pi_1(X^1, x_0) \twoheadrightarrow \pi_1(X, x_0).$$

   (b) Show that if $n \geq 2$ then $\pi_1(S^1 \vee S^n) \cong \mathbb{Z}$.

4. Find a space having the cyclic group C_3 of order 3 as its fundamental group.