Math 651 - Topology II
Homework IV
Spring 2014

1. Show that if $p : E \longrightarrow B$ is a covering and $A \subseteq B$ is any subspace, then the restriction of p gives a covering $p^{-1}(A) \longrightarrow A$.

2. (a) Show that any covering map $p : E \longrightarrow B$ is a quotient map.
   (b) Show that if E is very connected, so is B.
   (c) Suppose that E is very connected. Show that if $p^{-1}(b_0)$ has k elements for some $b_0 \in B$, this must be true for all $b \in B$. Such a covering is called a k**-sheeted covering** of B.

3. (a) Show that if $p_1 : E_1 \longrightarrow B_1$ and $p_2 : E_2 \longrightarrow B_2$ are covering maps, then so is $p_1 \times p_2 : E_1 \times E_2 \longrightarrow B_1 \times B_2$.
   (b) Use this to compute $\pi_1(T^2)$. (Of course, we already know the answer from problem 2 of Homework II.)

4. Suppose G acts on a space X. We say the action is **free** if $gx = x$ only for $g = e$. We say the action is **proper discontinuous** if every $x \in X$ has a neighbohood U such that U meets $g(U)$ only for finitely many $g \in G$.

   Show that if G acts freely and properly disontinuously on a Hausdorff space X, then $X \longrightarrow X/G$ is a covering map.

5. ($\star$) Find a simply connected covering of $\mathbb{RP}^2 \vee \mathbb{RP}^2$.