Math 651 - Topology II
Homework V
Spring 2014

1. Show that an injective covering map is a homeomorphism.

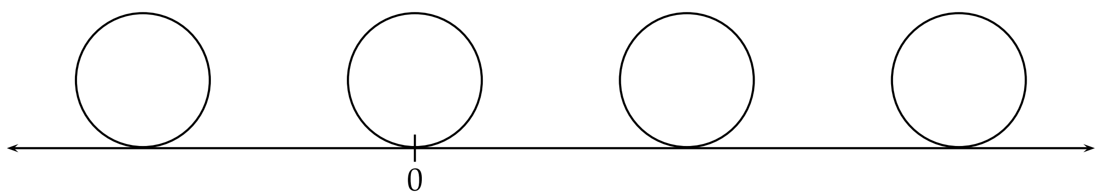
2. Suppose that $p : E \longrightarrow B$ is a nonsurjective covering map. That is, it satisfies the neighborhood condition but is not surjective. Show that the $p(E) \subseteq B$ is closed and open.

3. Let $p : E \longrightarrow B$ be a covering. For a space Z, show that the induced map

$$\mathrm{Map}(Z, p) : \mathrm{Map}(Z, E) \longrightarrow \mathrm{Map}(Z, B)$$

   is a (possibly nonsurjective) covering map. You may assume all spaces are locally compact Hausdorff.

4. (a) Describe a covering of $S^1 \vee S^1$ by the space E given in the picture below:



   (b) Take the point labelled as 0 as the basepoint for E. What is the image under your covering map p of the loop around the circle at 0? What about the loop (at 0) around the circle at 1?

   (c) Show that the two loops in E described in part (2) are not homotopic. Use this to show that $\pi_1(S^1 \vee S^1)$ is not abelian.

5. Let E_1 and E_2 be simply connected coverings of B_1 and B_2, respectively. Show that if $B_1 \simeq B_2$ then $E_1 \simeq E_2$. As usual, you may assume all spaces are very connected.

6. ($\star$) Let $p : E \longrightarrow B$ be a covering map. Show that any covering homomorphism from (E, p) to itself must be a homeomorphism.