Math 651 - Topology II
Homework VII
Spring 2014

1. (a) Show that in the Hawaiian earring (Example 22.1 in the notes), the point $(0,0)$ is **not** a nondegenerate basepoint. In other words, show that no neighborhood of $(0,0)$ deformation retracts onto $(0,0)$.

   (b) If no basepoint of X is nondegenerate, there is a way of adding in a good basepoint without changing the homotopy type. The process, known as "attaching a whisker to X", is to consider the space $X \vee I$. If we glue I to X along the point $0 \in I$, show that $1 \in I$ is a nondegenerate baspoint for $X \vee I$.

   (c) Show that $X \vee I$ is homotopy equivalent to X.

2. Use the van Kampen theorem to show that S^n is simply-connected if $n \geq 2$.

3. Let x and y be any two (distinct) points in $\mathbb{R}^3$. Use the van Kampen theorem to compute $\pi_1(\mathbb{R}^3 - \{x, y\})$.

4. Let X be $\mathbb{R}^3$ with two of the coordinate axes removed. Compute $\pi_1(X)$. (**Hint:** Start by showing that X is homotopy equivalent to S^2 with four points removed.)

5. $(\star)$ Let $X \subseteq \mathbb{R}^3$ be the union of S^2 and a diameter through the north and south poles. Find $\pi_1(X)$.