Math 651 - Topology II
Homework VIII
Spring 2014

1. Find $\pi_1(\mathbb{RP}^2 - \{x, y\})$, where $x \neq y$. (Hint: First find $\pi_1(\mathbb{RP}^2 - \{z\})$.)

2. (Möbius band) Let M be the quotient of I^2 obtained by identifying *one* pair of opposite edges via a twist (no glueing is performed on the other edges).

   (a) Show that $M \simeq S^1$.

   (b) Describe a CW structure on M and find $\chi(M)$.

   (c) Your CW structure on M in part (b) should give you a description of $\pi_1(M)$. How does this agree with (a)?

3. Let X be the quotient of S^2 obtained by identifying the north and south poles to a single point. Put a CW structure on X and use this to compute $\pi_1(X)$.

4. Give a purely algebraic argument to show that the groups with presentations

$$G_1 = \langle a, b \mid abab^{-1} \rangle, \qquad G_2 = \langle c, d \mid c^2 d^2 \rangle$$

   are isomorphic.

5. ($\star$) Let X and Y be finite CW complexes.

   (a) Use the cell structures on X and Y to put a cell structure on $X \times Y$. (Hint: It may help to use that $S^{m+n-1} \cong (S^{m-1} \times D^n) \cup_{S^{m-1} \times S^{n-1}} (D^m \times (S^{n-1}))$.) Don't worry about proving the (C) and (W) properties. The discussion in Problem 9.5.2 from last semester may be helpful.

   (b) Use this to deduce a formula for $\chi(X \times Y)$ in terms of $\chi(X)$ and $\chi(Y)$.