Math 651 - Topology II Homework IX Spring 2014

- 1. Let M_g be the surface of genus g, and let $x \in M_g$. Find $\pi_1(M_g \{x\})$.
- 2. (Problem 1.2.9 from Hatcher)
 - (*a*) Let M_g be the surface of genus *g*, with $g \ge 2$. Recall that

$$M_g \cong T^2 \# M_{g-1} = (T^2)' \cup_{S^1} M'_{g'1}$$

where the prime notation means that an open disk has been removed. Denote by $C \subseteq M_g$ the circle along which T^2 and M_{g-1} have been glued. Show that $(T_2)'$ and M'_{g-1} do not retract onto *C*. (Hint: Think about the abelianized fundamental groups.)

(*b*) Show that $(T_2)'$ and M_g do retract onto $C_2 \subseteq (T_2)'$.



- 3. In the following examples, compute the euler characteristic and determine the surface type. (Be careful—don't assume that these have a single 0-cell).
 - (*a*) X_1 has three 1-cells: *a*, *b*, and *c*; and a single 2-cell attached by $abacb^{-1}c^{-1}$.
 - (*b*) X_2 has three 1-cells: *a*, *b*, and *c*; and a single 2-cell attached by $abca^{-1}b^{-1}c^{-1}$.
 - (c) X_3 has four 1-cells: *a*, *b*, *c*, and *d*; and a single 2-cell attached by $abacdbd^{-1}c$.
 - (*d*) X_4 has four 1-cells: *a*, *b*, *c*, and *d*; and a single 2-cell attached by $abcdad^{-1}cb^{-1}$.
 - (e) X_5 has six 1-cells: *a*, *b*, *c*, *d*, *e*, and *f*; and four 2-cells attached by *abc*, *bde*, $c^{-1}df$, and $e^{-1}fa$.
- 4. Show directly that your answer for problem 3(b) is correct by cutting-and-pasting.