Math 751 - Topics in Topology
Homework 2
Spring 2015

1. We know that the defining quotient map $S^1 \longrightarrow \mathbb{RP}^1 \cong S^1$ is a double cover. Describe this as a (principal) $\mathbb{Z}/2\mathbb{Z}$-bundle.

2. Let $M = I^2/\sim$ be the Möbius band, where $(0, t) \sim (1, 1 - t)$. Show that projection onto the first coordinate defines a $\mathbb{Z}/2\mathbb{Z}$-bundle $M \longrightarrow S^1$. Find a local trivialization and describe the transition functions.

3. Describe the transition functions for the tautological bundle on the Grassmannian $\mathrm{Gr}_k(\mathbb{R}^n)$.

4. Describe the $O(k)$-bundle $V_k(\mathbb{R}^n) \longrightarrow \mathrm{Gr}_k(\mathbb{R}^n)$. That is, give the local trivializations and show that this defines a principal bundle.