Math 752 - Hopf algebras
Homework 1
Spring 2017

1. Show that in the bialgebra $\mathbf{F}_2[x_1]$, where $\deg(x_1) = 1$, then x^n is primitive if and only if n is a power of 2.

2. Consider the bialgebra $\mathbb{Z}[x_1]$, where $\deg(x_1) = 1$. (Note that this is **not** graded-commutative!) Show that x_1^2 is primitive.

3. Given that $SO(4) \cong \mathbb{RP}^3 \times S^3$ (see, for example, Hatcher, section 3D), compute the bialgebra structure on $\mathrm{H}^*(SO(4); \mathbf{F}_2)$ and $\mathrm{H}^*(SO(4); \mathbb{Q})$. Dualize these to deduce the bialgebra structure on the homology as well.

4. Consider $k\langle x, g\rangle / (x^2, g^2 = 1, xg + gx)$, where $\mathrm{char}(k) \neq 2$. Set

$$\begin{aligned} \Delta(g) &= g \otimes g, & \varepsilon(g) &= 1, \\ \Delta(x) &= x \otimes 1 + 1 \otimes x, & \varepsilon(x) &= 0. \end{aligned}$$

   Show that this defines a bialgebra. Note that it is not primitively generated, and no choice of grading will make it commutative or connected.