Math 752 - Hopf algebras
Homework 2
Spring 2017

1. Let C be an R-coalgebra and A and R-algebra. Show that the convolution product makes $\mathrm{Hom}(C, A)$ into an R-algebra with unit $\eta\varepsilon$.

2. Show that if A is commutative, then $\chi^2 = \mathrm{id}$. (Hint show that $\chi^2 * \chi = \eta\varepsilon$.)

3. Consider $k\langle x, g\rangle/(x^2, g^2 = 1, xg + gx)$, where $\mathrm{char}(k) \neq 2$. Set

$$\Delta(g) = g \otimes g, \qquad \varepsilon(g) = 1,$$
$$\Delta(x) = x \otimes 1 + g \otimes x, \qquad \varepsilon(x) = 0.$$

   This defines a bialgebra which is neither commutative nor cocommutative (you do not need to show this). Show that this has an antipode making it a Hopf algebra, but such that $\chi^2 \neq \mathrm{id}$.

4. Consider the commutative $\mathbf{F}_2$-bialgebra $B = \mathbf{F}_2[x_1, x_3]/(x_1^4, x_3^2)$, where $\deg(x_i) = i$, such that $\Delta(x_3) = x_3 \otimes 1 + x_1^2 \otimes x_1 + 1 \otimes x_3$. Compute the antipode on B, and describe the dual Hopf algebra.