Math 752 - Hopf algebras
Homework 4
Spring 2017

1. (a) Show by induction that $z_1^{2^k} = \boxed{2^k}^\vee$.

   (b) Let $J_2 \subseteq A_*$ be the Hopf ideal $J_2 = (z_1^8, z_2^4, z_3^2, z_4, z_5, \dots)$. Then $A(2) \subseteq A$ is, by definition, the subset of elements which vanish on J_2.

   Show that $\boxed{1}$, $\boxed{2}$, and $\boxed{4}$ vanish on (the generators of) J_2 and are therefore in $A(2)$. Show also that $\boxed{2^k}$, for $k \geq 3$, is not in $A(2)$.

2. Problem 3 on Worksheet 2 introduced the idea of "stripping" by z_1 to deduce Adem relations. Convince yourself that stripping by z_1^2 or z_1^4 (or $z_1^{2^k}$) works just as well. Use this to compute $\boxed{6}^2$ and $\boxed{6}^3$.

3. Compute the cohomology $\mathrm{H}^*(\mathbf{F}_2[x_1]/x_1^4)$. (Hint: build a periodic resolution.)