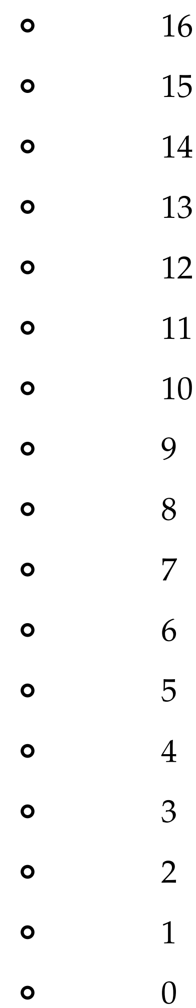
Math 752 - Hopf algebras
Worksheet 1
Spring 2017

1. If $p(x) \in \mathbf{F}_2[x]$, we define the "total Steenrod square" on $p(x)$ to be

$$\boxed{T}\,(p(x)) := \sum_{k \geq 0} \boxed{k}\,(p(x)).$$

   (a) Show that the total Steenrod square gives a ring homomorphism $\boxed{T} : \mathbf{F}_2[x] \longrightarrow \mathbf{F}_2[x]$.

   (b) Using only your knowledge of $\boxed{T}\,(x)$, compute $\boxed{T}\,(x^n)$, and deduce the formula $\boxed{k}\,x^n = \binom{n}{k} x^{n+k}$.



2. On the diagram to the right, draw in the nontrivial actions of $\boxed{1}$, $\boxed{2}$, $\boxed{4}$. Can you describe, in general, which powers of x support a $\boxed{2^k}$? Can you describe when $\boxed{5}$ or $\boxed{6}$ act nontrivially?

3. Define $E(1) \subseteq \mathcal{A}$ to be the subalgebra generated by $Q_0 := \boxed{1}$ and $Q_1 := \boxed{3} + \boxed{2,1}$. Show that $E(1) \cong E(Q_0, Q_1)$ is an exterior (commutative) algebra.

4. There is an operation referred to as "stripping", which allows you to get away with remembering only the single family of Adem relations $\boxed{2n-1}\boxed{n} = 0$. The stripping procedure says that from the trivial product $\boxed{2n-1}\boxed{n}$, you can "strip" one from each square in turn, and retain a valid relation. Thus $\boxed{3}\boxed{2} = 0$ implies that $\boxed{2}\boxed{2} + \boxed{3}\boxed{1} = 0$. Use stripping to compute $\boxed{8}\boxed{6}$.