Math 752 - Hopf algebras
Worksheet 2
Spring 2017

1. Let $E(1)_* = \mathcal{A}_*/(z_1^2, z_2^2, z_3, z_4, \dots) = E(z_1, z_2)$. This is a quotient Hopf algebra of $\mathcal{A}_*$. Let $E(1)$ be the dual of $E(1)_*$, with $Q_0 = z_1^\vee$ and $Q_1 = z_2^\vee$. We have Hopf maps

$$\mathcal{A}_* \twoheadrightarrow E(1)_*, \qquad E(1) \hookrightarrow \mathcal{A}.$$

   Express the image of $E(1)$ in the admissible basis, and show that $E(1) \cong E(Q_0, Q_1)$ is an exterior (commutative) algebra. Furthermore, show that Q_0 and Q_1 are primitive.

2. Let $A(1)_* = \mathcal{A}_*/(z_1^4, z_2^2, z_3, z_4, \dots) = \mathbb{F}_2[z_1, z_2]/(z_1^4, z_2^2)$. This is a quotient Hopf algebra of $\mathcal{A}_*$. Let $A(1)$ be the dual of $A(1)_*$. We have Hopf maps

$$\mathcal{A}_* \twoheadrightarrow A(1)_*, \qquad A(1) \hookrightarrow \mathcal{A}.$$

   Show that $(z_1^n)^\vee = \boxed{n}$ for $n = 1, 2, 3$, and more generally express the image of $A(1)$ in the admissible basis.

3. There is an operation referred to as "stripping", which allows you to get away with remembering only the single family of Adem relations $\boxed{2n-1}\boxed{n} = 0$. The idea is to consider the composition

$$\mathcal{A}_* \otimes \mathcal{A} \xrightarrow{\mathrm{id} \otimes \Delta} \mathcal{A}_* \otimes \mathcal{A} \otimes \mathcal{A} \xrightarrow{\mathrm{eval} \otimes \mathrm{id}} \mathbb{F}_2 \otimes \mathcal{A}.$$

   The stripping procedure says that since $\boxed{2n-1}\boxed{n} = 0$, it follows that this composition sends $z_1 \otimes \boxed{2n-1}\boxed{n} \mapsto 0$. So evaluating this image gives a new relation in $\mathcal{A}$.

   Thus $\boxed{3}\boxed{2} = 0$ implies that $\boxed{2}\boxed{2} + \boxed{3}\boxed{1} = 0$. The stripping procedure says that from the trivial product $\boxed{2n-1}\boxed{n}$, you can "strip" one from each square in turn, and retain a valid relation. Use stripping to compute $\boxed{8}\boxed{6}$.