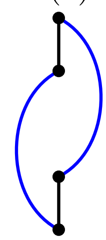
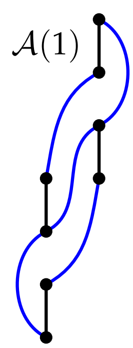
Math 752 - Hopf algebras
Worksheet 5
Spring 2017

$\mathcal{E}(1)$

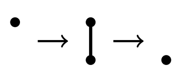
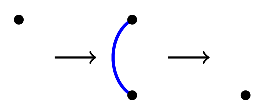


1. Build the first 4 stages (i.e., up to P_3) of a minimal resolution of k over $E(x_1, x_3)$. Verify that the resulting vector space dimensions of $\mathrm{Ext}^n_{E(x_1,x_3)}(k,k)$ agree with $k[v_0, v_1]$ for $n \leq 3$.

$\mathcal{A}(1)$



2. Recall that $\mathcal{A}(1) \subseteq \mathcal{A}$ is the subalgebra generated by $\boxed{1}$ and $\boxed{2}$. The algebra generators $\boxed{1}$ and $\boxed{2}$ give rise to elements $h_0 \in \mathrm{Ext}^{1,1}_{\mathcal{A}(1)}$ and $h_1 \in \mathrm{Ext}^{1,2}_{\mathcal{A}(1)}$ which can be thought of as the extensions.

h_0 :  and h_1 : 

   (a) Show that $h_0h_1 = 0$ in $\mathrm{Ext}^{2,3}_{\mathcal{A}(1)}$ using the extension approach.

   (b) Show that $h_0h_1 = 0$ in $\mathrm{Ext}^{2,3}_{\mathcal{A}(1)}$ from the cobar complex. Recall that $\mathcal{A}(1)^\vee \cong \mathbf{F}_2[z_1, z_2]/(z_1^4, z_2^2)$, with z_1 primitive and $\Delta(z_2) = z_1^2 \otimes z_1$.