Math 651 - Topology II
Homework III
Spring 2018

1. (a) Let X be a finite CW complex (meaning finitely many cells), and let x_0 be a point in the 0-skeleton. Modify the proof that S^n is simply-connected for $n \geq 2$ to show that the inclusion of the 1-skeleton $X^1 \hookrightarrow X$ induces a surjection

$$\pi_1(X^1, x_0) \twoheadrightarrow \pi_1(X, x_0).$$

   (b) Show that if $n \geq 2$ then $\pi_1(S^1 \vee S^n) \cong \mathbb{Z}$.

2. Show that if $p : E \longrightarrow B$ is a covering and $A \subseteq B$ is any subspace, then the restriction of p gives a covering $p^{-1}(A) \longrightarrow A$.

3. Here's some terminology I made up: X is "very connected" if it is connected and locally path-connected.

   (a) Show that any covering map $p : E \longrightarrow B$ is a quotient map.

   (b) Show that if E is very connected, so is B.

   (c) Suppose that E is very connected. Show that if $p^{-1}(b_0)$ has k elements for some $b_0 \in B$, this must be true for all $b \in B$. Such a covering is called a **k-sheeted covering** of B.

4. (a) Show that if $p_1 : E_1 \longrightarrow B_1$ and $p_2 : E_2 \longrightarrow B_2$ are covering maps, then so is $p_1 \times p_2 : E_1 \times E_2 \longrightarrow B_1 \times B_2$.

   (b) Use this to compute $\pi_1(T^2)$. (Of course, we already know the answer from problem 2 of Homework II.)