Math 651 - Topology II
Homework IV
Spring 2018

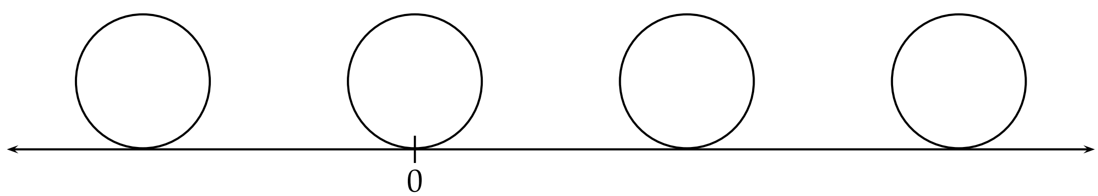
1. Suppose that $p : E \longrightarrow B$ is a nonsurjective covering map. That is, it satisfies the neighborhood condition but is not surjective. Show that the $p(E) \subseteq B$ is closed and open.

2. Suppose G acts on a space X. We say the action is **free** if, for every x, we have $gx = x$ only for $g = e$. We say the action is **properly discontinuous** if every $x \in X$ has a neighbohood U such that U meets $g(U)$ only for finitely many $g \in G$.

   Show that if G acts freely and properly disontinuously on a Hausdorff space X, then $X \longrightarrow X/G$ is a covering map.

3. Find a space having fundamental group C_3, the cyclic group of order three.

4. (a) Describe a covering of $S^1 \vee S^1$ by the space E given in the picture below:



   (b) Take the point labelled as 0 as the basepoint for E. What is the image under your covering map p of the loop around the circle at 0? What about the loop (at 0) around the circle at 1?

   (c) Show that the two loops in E described in part (2) are not homotopic. Use this to show that $\pi_1(S^1 \vee S^1)$ is not abelian.

5. Let E_1 and E_2 be simply connected coverings of B_1 and B_2, respectively. Show that if $B_1 \simeq B_2$ then $E_1 \simeq E_2$. As usual, you may assume all spaces are very connected.

6. ($\star$) Find a simply connected covering of $\mathbb{RP}^2 \vee \mathbb{RP}^2$.