Math 651 - Topology II
Homework V
Spring 2018

1. Let $q : X \longrightarrow B$ be a simply-connected covering. We showed in class that $\mathrm{Aut}(X) \cong G = \pi_1(B)$. This gives a left action of G on X, and this action restricts to an action on any fiber. But we also discussed a right G-action on the fiber for any covering.

   (a) Let $q : \mathbb{R}^2 \longrightarrow S^1 \times S^1$ be the universal covering of the torus. Show the two above actions are the same.

   (b) Let $q : X \longrightarrow S^1 \vee S^1$ be the (fractal) simply-connected covering discussed in class. Show that in this case the two actions *do not* coincide! (Hint: Denote by α and β the loops around the two circles in $S^1 \vee S^1$. Determine (carefully) the action of $\alpha\beta$ on a point in the fiber under the two described actions.)

2. Use the van Kampen theorem to show that S^n is simply-connected if $n \geq 2$.

3. Show that if $x_0 \in X$ is a 0-cell for a finite CW structure on X, then x_0 is a nondegenerate basepoint. (The statement is true more generally without the finiteness hypothesis).

4. (a) Show that in the Hawaiian earring (Example 2.26 in the notes), the point $(0,0)$ is **not** a nondegenerate basepoint. In other words, show that no neighborhood of $(0,0)$ deformation retracts onto $(0,0)$.

   (b) If no basepoint of X is nondegenerate, there is a way of adding in a good basepoint without changing the homotopy type. The process, known as "attaching a whisker to X", is to consider the space $X \vee I$. If we glue I to X along the point $0 \in I$, show that $1 \in I$ is a nondegenerate baspoint for $X \vee I$.

   (c) Show that $X \vee I$ is homotopy equivalent to X.