Math 651 - Topology II
Homework VI
Spring 2018

1. Let x and y be any two (distinct) points in $\mathbb{R}^3$. Use the van Kampen theorem to compute $\pi_1(\mathbb{R}^3 - \{x, y\})$.

2. Let X be $\mathbb{R}^3$ with two of the coordinate axes removed. Compute $\pi_1(X)$. (**Hint:** Start by showing that X is homotopy equivalent to S^2 with four points removed.)

3. (a) Show that for any space A, the cone $C(A) = (A \times I)/(A \times \{1\})$ is contractible. (Hint: What happens in the case that A is empty?)

   (b) Let $f : A \longrightarrow X$ be a based map between path-connected spaces. Then form the union
   $$C(f) = X \cup_A C(A),$$
   where A is included in $C(A)$ at height 0. This construction is known as the mapping cone of f. Show that
   $$\pi_1(C(f)) \cong \pi_1(X)/N,$$
   where N is the normal subgroup generated by the image of $f_* : \pi_1(A) \longrightarrow \pi_1(X)$.

   (c) Let G be a finitely-generated group, meaning that there is a surjection $F_n \longrightarrow G$ for some n. Find a (path-connected) space X with $\pi_1(X) \cong G$.

4. Find $\pi_1(\mathbb{RP}^2 - \{x, y\})$, where $x \neq y$. (Hint: First find $\pi_1(\mathbb{RP}^2 - \{z\})$.)

5. Let X be the quotient of S^2 obtained by identifying the north and south poles to a single point. Put a CW structure on X and use this to compute $\pi_1(X)$.