Math 651 - Topology II
Homework IX
Spring 2018

1. (Not to be turned in) Recreate the proof of Proposition 5.23 on your own, without consulting the notes.

2. Recall that the **(unreduced) suspension** of X is $SX = CX \cup_X CX$.

   (a) Use the Mayer-Vietoris sequence to show that $\mathrm{H}_{n+1}(SX) \cong \mathrm{H}_n(X)$ for $n \geq 1$.

   (b) Suppose that X is nonempty, with finitely many path-components. Determine $\mathrm{H}_1(SX)$.

3. (a) Find the homology groups $\mathrm{H}_k(S^1 \vee S^1)$.

   (b) Find the homology groups $\mathrm{H}_k(\bigvee_n S^1)$ of the wedge of n circles.

4. Suppose that X is obtained from A by attaching an n-cell, where $n \geq 2$.

   (a) Show that there is an exact sequence

   $$0 \longrightarrow \mathrm{H}_n(A) \longrightarrow \mathrm{H}_n(X) \longrightarrow \mathbb{Z} \longrightarrow \mathrm{H}_{n-1}(A) \longrightarrow \mathrm{H}_{n-1}(X) \longrightarrow 0.$$

   (b) Show that $\mathrm{H}_k(A) \cong \mathrm{H}_k(X)$ if $k \geq 1$ and $k \neq n, n-1$.

5. (a) Find the homology groups $\mathrm{H}_k(T^2)$.

   (b) Find the homology groups $\mathrm{H}_k(\mathbb{RP}^2)$.