

Math 751 – Spring 2024

Chromatic Homotopy Theory

Worksheet 3

1. In this problem, you will verify the formula for $H^*(K(\mathbb{Z}, n); \mathbb{Q})$ given in class.

(a) Start by finding $H^k(K(\mathbb{Z}, n); \mathbb{Q})$ for $k \leq n$.

(b) Use induction and the Serre spectral sequence for the fiber sequence

$$K(\mathbb{Z}, n-1) \rightarrow PK(\mathbb{Z}, n) \rightarrow K(\mathbb{Z}, n)$$

to verify the formula for $H^*(K(\mathbb{Z}, n); \mathbb{Q})$.

2. Show that for any nonzero $n \in \mathbb{Z}$, the spectrum S/n is rationally acyclic.

3. Let $\text{Nul}_{\mathbb{Q}}X$ denote the fiber of $X \rightarrow X_{\mathbb{Q}}$. Show that $\text{Nul}_{\mathbb{Q}}X$ is rationally acyclic, for any X .