

# Math 751 – Spring 2024

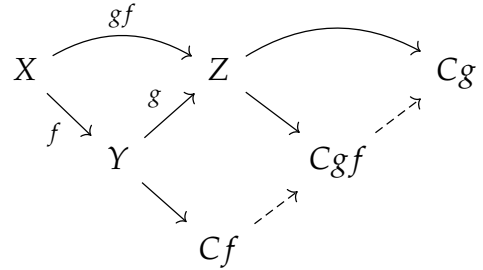
## Chromatic Homotopy Theory

### Worksheet 13

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1. Let  $X \xrightarrow{f} Y \rightarrow Z$  be a cofiber sequence. If  $f$  is zero in  $\mathbf{HoSp}$ , show that  $Z \simeq Y \vee \Sigma^1 X$ .
  
2. Let  $\mathcal{C} \subset \mathbf{HoSp}$  be a thick subcategory. Suppose that for some finite spectrum  $J$  and map  $J \xrightarrow{f} S$ , the cofiber  $Cf$  is in  $\mathcal{C}$ . Use induction to show that the cofiber of  $J^{\wedge k} \xrightarrow{f^{\wedge k}} S$  is also in  $\mathcal{C}$  for each  $k \geq 1$ .

(Hint: The octahedral axiom, also known as the braid axiom, says that given the solid cofiber sequences in the diagram to the right, the dashed sequence is also a cofiber sequence. This is analogous to the isomorphism theorem  $(G/H)/(K/H) \cong G/K$ .)



3. Let  $f: \Sigma^d X \rightarrow X$  be a self-map. Show that

$$\langle X \rangle = \langle X/f \rangle \vee \langle f^{-1}X \rangle.$$

(Hint: For the direction  $\supset$ , show that if  $Z$  is  $X/f$ -acyclic, then  $X_*Z \cong f^{-1}X_*Z$ .)