

Math 751 – Spring 2024

Chromatic Homotopy Theory

Worksheet 9

1. Fix a prime p . The Quillen idempotent $e: MU_{(p)} \rightarrow MU_{(p)}$ acts on the elements $[\mathbb{CP}^n] \in MU_{2n}$ according to

$$e(\mathbb{CP}^n) = \begin{cases} \mathbb{CP}^n & n = p^k - 1, \text{ some } k \\ 0 & \text{else.} \end{cases}$$

Modulo decomposable elements, \mathbb{CP}^{p^k-1} is p^{k-1} times a generator. For example, \mathbb{CP}^8 is 3 times a generator. Use this information to deduce the homotopy groups of the ring spectrum $BP = e^{-1}MU_{(p)}$.

2. Assuming, as stated in class, that $MU_{(p)}$ splits as a wedge of suspensions of BP , show that there must be **infinitely many** summands in this wedge. (Hint: consider $p = 3$, for example).