The Homotopy of Normed Mackey Functors

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March 2026

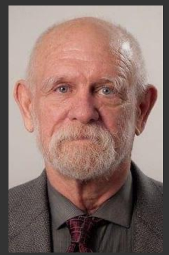
The homotopy of normed Mackey functors

1. Background
2. Norms
3. Coefficients

1 Background & Motivation

$\pi_* \mathbb{S}$ stable homotopy groups of spheres

Chromatic approach: stratify p-locally by height



Height n detected by

$$\mathbb{S} \longrightarrow L_{K(n)}\mathbb{S}$$

Ex $n=0$

$$\mathbb{S} \longrightarrow H\mathbb{Q}$$

Ex $n=1$

$$\mathbb{S} \longrightarrow L_{K(1)}\mathbb{S} \quad \sim \mathrm{im}\, J$$

③

[Devinatz-Hopkins] $L_{K(n)}\mathbb{S} \simeq E_n^{h\mathbb{G}_n}$

Morava stabilizer group

Morava E-theory a.k.a. Lubin-Tate spectrum

$\implies L_{K(n)}\mathbb{S}$?? can be computed by HFP spectral sequence

$$E_2 = H^*(\mathbb{G}_n ; \pi_* E_n) \Rightarrow \pi_* L_{K(n)}\mathbb{S}$$

Problem $\mathbb{G}_n \circlearrowright \pi_* E_n$ mysterious

Instead, consider finite $H \leq \mathbb{G}_n$, use E_n^{hH} to approximate $E_n^{h\mathbb{G}_n} \simeq L_{K(n)}\mathbb{S}$.

②

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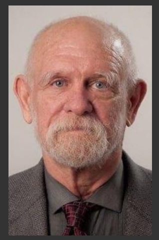

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④

Ex $p=2$, $n=1$

$E_1 = KU_2^\wedge$, $\mathbb{G}_1 = \mathbb{Z}_2^\times \cong C_2 \times \mathbb{Z}_2$

$E_1^{h\mathbb{G}_1} = ((KU_2^\wedge)^{hC_2})^{h\mathbb{Z}_2} \simeq (KO_2^\wedge)^{h\mathbb{Z}_2}$

$KR_2^\wedge$

$\Longrightarrow$ $L_{K(1)}\mathbb{S} = \mathrm{hofib}\left(KO_2^\wedge \xrightarrow{\psi^3 - \mathrm{Id}} KO_2^\wedge\right)$

Higher heights?

[Hewett] Write $n = 2^k \cdot \text{odd}$

The maximal 2-subgroup $H \leq \mathbb{G}_n$ is $\begin{cases} C_{2^{k+1}} & \text{if } k \neq 1 \\ Q_8 & \text{if } k = 1 \end{cases}$

[Hill-Hopkins-Ravenel]

Slice spectral sequence for $MU\mathbb{R}$

& $N_{C_2}^{C_{2^k}} MU\mathbb{R}$.

[Hahn-Shi] $MU\mathbb{R} \longrightarrow E_n$,

computed HFPSS for $E_n^{hC_2}$ $\forall n$.

[Hill-Shi-Wang-Xu]

Slice SS for $N_{C_2}^{C_4} \overbrace{MU\mathbb{R}}^{BP\mathbb{R}}$

$\rightsquigarrow E_4^{hC_{12}}$ is 384-periodic

[Duan-Kong-Li-Lu-Wang]

computed HFPSS for $E_2^{hQ_8}$

Ex $p=2,\ n=1$

$E_1 = KU_2^\wedge,\quad \mathbb{G}_1 = \mathbb{Z}_2^\times \cong C_2 \times \mathbb{Z}_2$

$E_1^{h\mathbb{G}_1} = \left((KU_2^\wedge)^{hC_2}\right)^{h\mathbb{Z}_2} \simeq (KO_2^\wedge)^{h\mathbb{Z}_2}$

$K\mathbb{R}_2^\wedge$

$\Rightarrow L_{K(1)}\mathbb{S} = \mathrm{hofib}\left(KO_2^\wedge \xrightarrow{\psi^3 - \mathrm{Id}} KO_2^\wedge\right)$

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(6)

(2) Norms

$H \leq G$

$$N_H^G : Sp^H \longrightarrow Sp^G$$

$$Top^H \longrightarrow Top^G$$

$$X \longmapsto Map_H(G, X)$$

$$= Map_H\Big(\coprod_{H\backslash G} Hg, X\Big)$$

$$= \prod_{H\backslash G} X$$

For $H \trianglelefteq G$ normal, have

$$(N_H^G(X))^H = \prod_{H\backslash G} X^H \in Top^{G/H}$$

$$= N_e^{G/H} X^H$$

[Meier-Shi-Zeng]

⑦

Also works in spectra:

$$\Phi^H N_H^G X \simeq N_e^{G/H} \Phi^H X \quad \text{in } Sp^{G/H}$$

$$\Rightarrow \Phi^{C_2} N_{C_2}^{C_4} BP\mathbb{R} \simeq N_e^{C_2} \Phi^{C_2} BP\mathbb{R}$$

$$\simeq N_e^{C_2} H\mathbb{F}_2$$

Aside: E_2-term for HFP ss $\Rightarrow (N_e^{C_2} H\mathbb{F}_2)^{hC_2}$

is $H^*(C_2; \pi_* N_e^{C_2} H\mathbb{F}_2) = H^*(C_2; \mathcal{A}_*)$

dual Steenrod, action via antipode

Not known!

⑥

2

Norms

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Not known!

⑧

Similarly, can consider $N_{C_2}^{Q_8} BP\mathbb{R}$

$$\begin{tikzcd}[column sep=small] & Q_8 \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\ C_4 \arrow[dr, dash] & C_4 \arrow[d, dash] & C_4 \arrow[dl, dash] \\ & C_2 \arrow[d, dash] & \\ & e & \end{tikzcd} \qquad \begin{tikzcd}[column sep=small] & K_4 \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\ C_2 \arrow[dr, dash] & C_2 \arrow[d, dash] & C_2 \arrow[dl, dash] \\ & e & \end{tikzcd}$$

$$\Phi^{C_2} N_{C_2}^{Q_8} BP\mathbb{R} \simeq N_e^{K_4} H\mathbb{F}_2 \qquad \text{Difficult}$$

Simpler: study the truncation

$$H_K \underline{\pi}_0(\;) \cong H_K N_e^{K_4} \mathbb{F}_2$$

Mackey norm [Hoyer, Mazur]

$$N_H^G : \mathrm{Mack}(H) \longrightarrow \mathrm{Mack}(G)$$

Ex $N_e^G \mathbb{Z} \cong \underline{A}$ Burnside

$$N_e^{C_2}\mathbb{Z} = \underline{A}_{C_2} \qquad \begin{tikzcd} \mathbb{Z}\{C/C, C/e\} \arrow[d, bend right, "(1\ 2)"'] \\ \mathbb{Z} \arrow[u, bend right, "\begin{pmatrix}0\\1\end{pmatrix}"'] \end{tikzcd}$$

$$N_e^{C_2}\mathbb{Z}/2 = \begin{tikzcd} \mathbb{Z}/4 \arrow[d, bend right, "1"'] \\ \mathbb{Z}/2 \arrow[u, bend right, "2"'] \end{tikzcd} \qquad N_e^{C_2}\mathbb{Z}/p = \begin{tikzcd} \mathbb{Z}/p\{C/C, C/e\} \arrow[d, bend right, "(1\ 2)"'] \\ \mathbb{Z}/p \arrow[u, bend right, "\begin{pmatrix}0\\1\end{pmatrix}"'] \end{tikzcd}$$

$$N_e^{K_4}\mathbb{Z} = \underline{A}_{K_4} = \begin{tikzcd}[column sep=small] & \mathbb{Z}\{K/K, K/L, K/D, K/R, K/e\} \arrow[dl, bend right, "\begin{pmatrix}1&2&0&0&0\\0&0&1&1&2\end{pmatrix}"'] \arrow[d, bend right, "\begin{pmatrix}1&0&2&0&0\\0&1&0&1&2\end{pmatrix}"'] \arrow[dr, bend right, "\begin{pmatrix}1&0&0&2&0\\0&1&1&0&2\end{pmatrix}"'] & \\ \mathbb{Z}\{L/L, L/e\} \arrow[ur, bend right, "\begin{pmatrix}0&0\\1&0\\0&0\\0&0\\0&1\end{pmatrix}"'] \arrow[dr, bend right, "(1\ 2)"'] & \mathbb{Z}\{D/D, D/e\} \arrow[u, bend right, "\begin{pmatrix}0&0\\0&0\\1&0\\0&0\\0&1\end{pmatrix}"'] \arrow[d, bend right, "(1\ 2)"'] & \mathbb{Z}\{R/R, R/e\} \arrow[ul, bend right, "\begin{pmatrix}0&0\\0&0\\0&0\\1&0\\0&1\end{pmatrix}"'] \arrow[dl, bend left, "(1\ 2)"] \\ & \mathbb{Z} \arrow[ul, bend right, "\begin{pmatrix}0\\1\end{pmatrix}"'] \arrow[u, bend right, "\begin{pmatrix}0\\1\end{pmatrix}"'] \arrow[ur, bend left, "\begin{pmatrix}0\\1\end{pmatrix}"] & \end{tikzcd}$$

Similarly, can consider $N_{C_2}^{Q_8} BP\mathbb{R}$

$$\begin{tikzcd}[column sep=tiny] & Q_8 \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\ C_4 \arrow[dr, dash] & C_4 \arrow[d, dash] & C_4 \arrow[dl, dash] \\ & C_2 \arrow[d, dash] & \\ & e & \end{tikzcd} \qquad \begin{tikzcd}[column sep=tiny] & K_4 \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\ C_2 \arrow[dr, dash] & C_2 \arrow[d, dash] & C_2 \arrow[dl, dash] \\ & e & \end{tikzcd}$$

$$\Phi^{C_2} N_{C_2}^{Q_8} BP\mathbb{R} \simeq N_e^{K_4} H\mathbb{F}_2$$

Simpler: study the truncation

$$H_K \underline{\pi}_0(\ ) \cong H_K N_e^{K_4}\mathbb{F}_2$$

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$$N_e^{K_4}\mathbb{Z} = \underline{A}_{K_4} =$$

$$\begin{tikzcd}[column sep=small] & \mathbb{Z}\{K/K, K/L, K/D, K/R, K/e\} \arrow[dl, shift right, "\begin{pmatrix}1&2&0&0&0\\0&0&1&1&2\end{pmatrix}"'] \arrow[d, shift right, "\begin{pmatrix}1&0&2&0&0\\0&1&0&1&2\end{pmatrix}"'] \arrow[dr, shift right, "\begin{pmatrix}1&0&0&2&0\\0&1&1&0&2\end{pmatrix}"'] & \\ \mathbb{Z}\{L/L, L/e\} \arrow[ur, shift right, "\begin{pmatrix}0&0\\1&0\\0&0\\0&0\\0&1\end{pmatrix}"'] \arrow[dr, shift right, "(1\ 2)"'] & \mathbb{Z}\{D/D, D/e\} \arrow[u, shift right, "\begin{pmatrix}0&0\\0&0\\1&0\\0&0\\0&1\end{pmatrix}"'] \arrow[d, shift right, "(1\ 2)"'] & \mathbb{Z}\{R/R, R/e\} \arrow[ul, shift right, "\begin{pmatrix}0&0\\0&0\\0&0\\1&0\\0&1\end{pmatrix}"'] \arrow[dl, shift right, "(1\ 2)"'] \\ & \mathbb{Z} \arrow[ul, shift right, "\binom{0}{1}"'] \arrow[u, shift right, "\binom{0}{1}"'] \arrow[ur, shift right, "\binom{0}{1}"'] & \end{tikzcd}$$

Theorem (G-Keyes-Mehrle)

$N_e^{K_4}\mathbb{F}_2$ is

$$\begin{tikzcd}[column sep=small] & \mathbb{Z}/8 \oplus \mathbb{F}_2\{b_L, b_R\} \arrow[dl, shift right] \arrow[d, shift right] \arrow[dr, shift right] & \\ \mathbb{Z}/4 \arrow[ur, shift right, "b_L - 2"'] \arrow[dr, shift right, "1"'] & \mathbb{Z}/4 \arrow[u, shift right, "b_L + b_R + 2"'] \arrow[d, shift right, "1"'] & \mathbb{Z}/4 \arrow[ul, shift right, "b_R - 2"'] \arrow[dl, shift right, "1"'] \\ & \mathbb{Z}/2 \arrow[ul, shift right, "2"'] \arrow[u, shift right, "2"'] \arrow[ur, shift right, "2"'] & \end{tikzcd}$$

restrictions are ring maps w/ $b_L, b_R \longmapsto 0$

$\{b_L = K/L + 2,\ b_R = K/R + 2\}$

Sketch $N_e^{K_4}$ is left adjoint to restriction for G-commutative monoids (Tambara functors).

$\Rightarrow N_e^{K_4}$ preserves pushouts.

Use pushout of rings

$$\begin{tikzcd} \mathbb{Z}[x] \arrow[r, "2"] \arrow[d, "0"'] & \mathbb{Z} \arrow[d] \\ \mathbb{Z} \arrow[r] & \mathbb{Z}/2 \arrow[ul, phantom, "\ulcorner", very near start] \end{tikzcd}$$

3 Coefficients

For $X \in Sp^G$, typically compute

$$\pi_\star X \quad \text{or} \quad \underline{\pi}_\star X$$

RO(G)-grading

$$RO(K_4) = \mathbb{Z}\{1, \underbrace{\sigma_L, \sigma_D, \sigma_R}_{\overline{g}}\}$$

Instead grade over $\blacklozenge \in \mathbb{Z}\{1, \overline{g}\} = RO(K_4)^{Aut(K)}$

Want

$$\underline{\pi}_{n+kg} H N_e^{K_4} \mathbb{Z}/2 \cong \underline{\pi}_0 \Sigma^{-n-kg} H N_e^{K_4} \mathbb{Z}/2$$

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 & \mathbb{Z}/8 \oplus \mathbb{F}_2\{b_L, b_R\} \arrow[dl, shift left, "b_L - 2"] \arrow[d, shift left, "b_L + b_R + 2"] \arrow[dr, shift left, "b_R - 2"] & \\
\mathbb{Z}/4 \arrow[ur, shift left] \arrow[dr, shift left, "1"] & \mathbb{Z}/4 \arrow[u, shift left] \arrow[d, shift left, "1"] & \mathbb{Z}/4 \arrow[ul, shift left] \arrow[dl, shift left, "1"] \\
 & \mathbb{Z}/2 \arrow[ul, shift left, "2"] \arrow[u, shift left, "2"] \arrow[ur, shift left, "2"] &
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Instead grade over

$$\diamond \in \mathbb{Z}\{1, \bar{\rho}\} = RO(K_4)^{Aut(K)}$$

Want

$$\underline{\pi}_{n+k\bar{\rho}} HN_e^{K_4}\mathbb{Z}/2$$

$$\cong \underline{\pi}_0 \Sigma^{-n-k\bar{\rho}} HN_e^{K_4}\mathbb{Z}/2$$

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Warmup case: $N = N_e^{C_2}\mathbb{Z}/2 = \begin{tikzcd} \mathbb{Z}/4 \arrow[d, bend right, "1"'] \\ \mathbb{Z}/2 \arrow[u, bend right, "2"'] \end{tikzcd}$

C_2-CW structure on $S^\sigma =$ 

⇝ cofiber sequence

$$\underset{\displaystyle H\uparrow_e^{C_2}\downarrow_e^{C_2} N}{\overset{\shortparallel}{C_{2+}\wedge HN}} \to HN \to \Sigma^\sigma HN$$

$$\begin{tikzcd}[column sep=large] & \text{ker} & & & \text{coker} \\ & 0 & \mathbb{Z}/2 \arrow[r, "2"] \arrow[d, bend right, "\Delta"'] & \mathbb{Z}/4 \arrow[d, bend right, "1"'] & \mathbb{Z}/2 \\ \underline{\pi}_0 & \mathbb{Z}/2 & \mathbb{Z}/2\{C_2/e\} \arrow[r, "\nabla"] \arrow[u, bend right, "\nabla"'] & \mathbb{Z}/2 \arrow[u, bend right, "2"'] & 0 \\ & \overline{\bullet} & & & \bullet \end{tikzcd}$$

$$\Longrightarrow \underline{\pi}_n \Sigma^\sigma HN \cong \begin{cases} \overline{\bullet} & n = 1 \\ \bullet & n = 0 \\ 0 & \text{else} \end{cases}$$

For K_4 use

$$K/L_+ \wedge HN \longrightarrow HN \longrightarrow \Sigma^{\sigma_L} HN$$

$$K/D_+ \wedge \Sigma^{\sigma_L} HN \longrightarrow \Sigma^{\sigma_L} HN \longrightarrow \Sigma^{\sigma_L+\sigma_D} HN$$

$$K/R_+ \wedge \Sigma^{\sigma_L+\sigma_D} HN \longrightarrow \Sigma^{\sigma_L+\sigma_D} HN \longrightarrow \Sigma^{\overline{\delta}} HN$$

$$\rightsquigarrow \underline{\pi}_n \Sigma^{\overline{\delta}} H_K N \cong \begin{cases} \overline{\mathbb{F}_2} & n=3 \\ \phi_{LDR}\overline{\odot} & n=1 \\ \odot & n=0 \end{cases}$$

Warmup case: $N = N_e^{C_2}\mathbb{Z}/2 = \begin{matrix}\mathbb{Z}/4 \\ 1\downarrow\ \uparrow 2 \\ \mathbb{Z}/2\end{matrix}$

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$$\begin{matrix} C_{2+}\wedge HN \longrightarrow HN \longrightarrow \Sigma^{\sigma} HN \\ \| \\ H\uparrow_e^{C_2}\downarrow_e^{C_2} N \end{matrix}$$

ker		coker
0	$\mathbb{Z}/2 \xrightarrow{2} \mathbb{Z}/4$	$\mathbb{Z}/2$
	$\Delta\downarrow\ \uparrow\nabla \qquad 1\downarrow\ \uparrow 2$	
$\mathbb{Z}/2$	$\mathbb{Z}/2\{C_2/e\} \xrightarrow{\nabla} \mathbb{Z}/2$	0
$\overline{\bullet}$		$\bullet$

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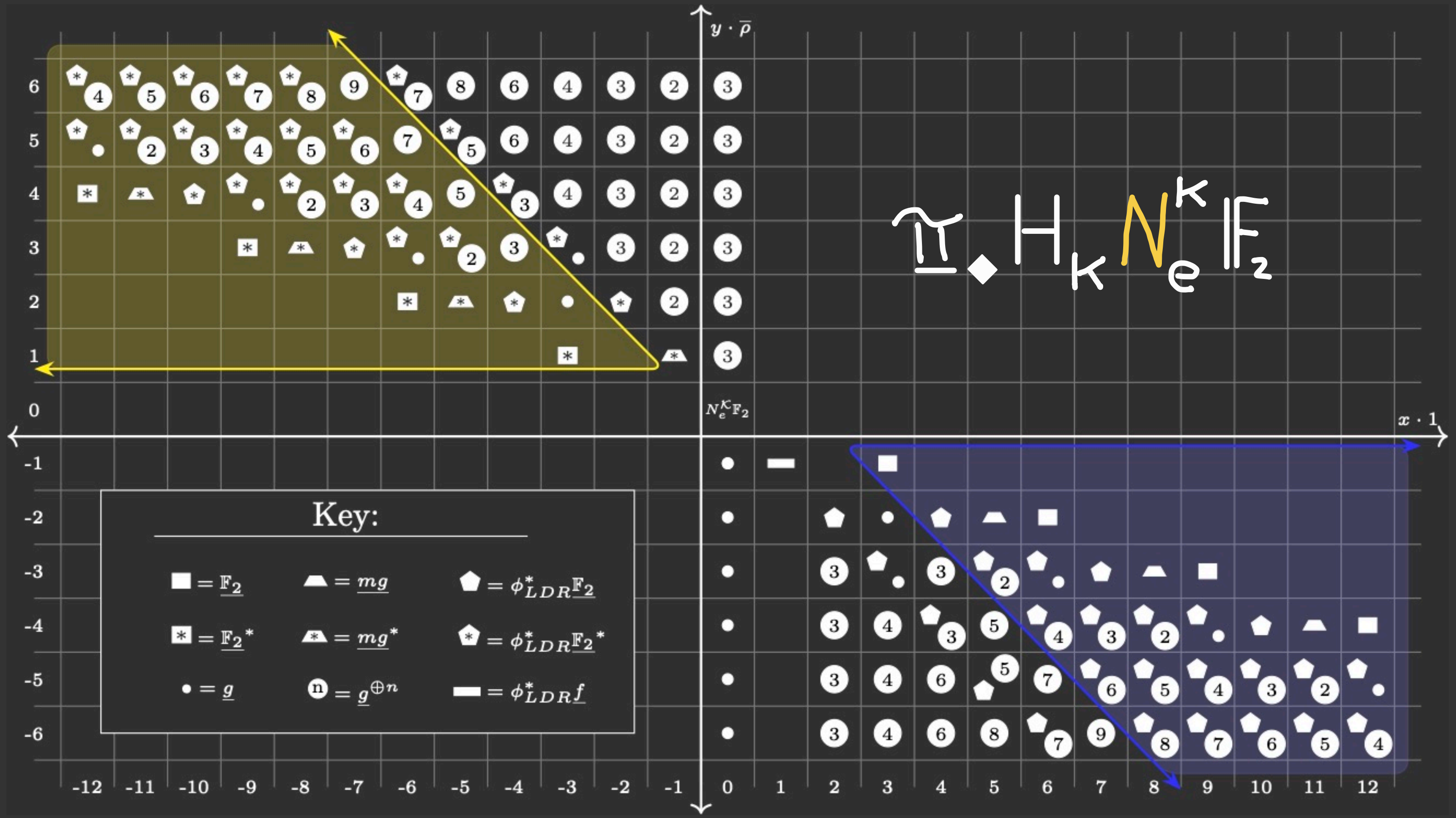
Theorem (G-Keyes-Mehrle)

$\underline{\pi}_\blacklozenge H_K N_e^K \mathbb{F}_2$ is



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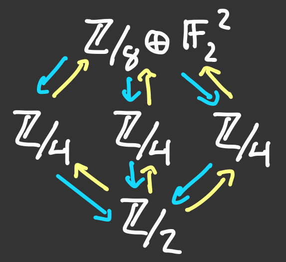
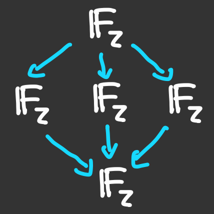
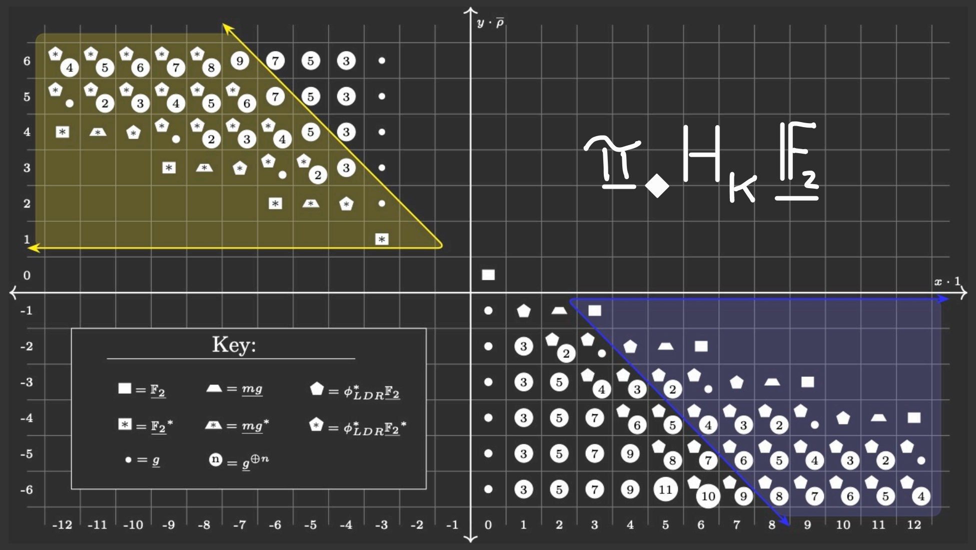
Sketch use $N_e^K \mathbb{F}_2 \longrightarrow \underline{\mathbb{F}_2}$



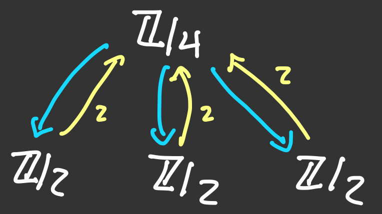


[G-Yarnall, Holler-Kriz]

$\underline{\pi}_\blacklozenge H_K \underline{\mathbb{F}_2}$ is



Kernel is



$\mathbb{F}_2\{b_L, b_R\}$

$\oplus$ O O O

O O

Compute $\underline{\pi}_\blacklozenge H_K \underline{\text{kernel}}$ & use LES.

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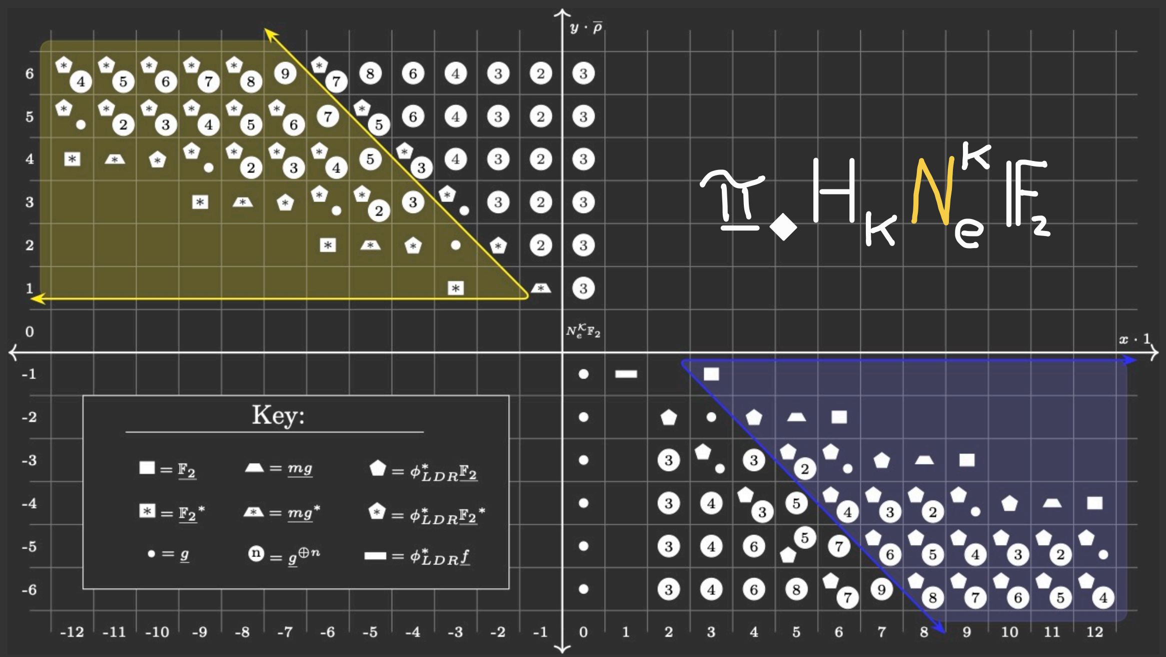
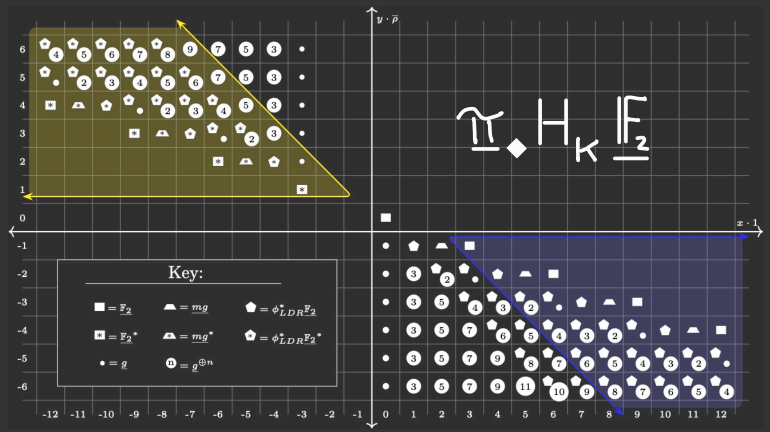
$\underline{\pi}_\blacklozenge H_K N_e^K \mathbb{F}_2$ is



Sketch use $\underline{N}_e^K \underline{\mathbb{F}}_2 \longrightarrow \underline{\mathbb{F}}_2$

[G-Yarnall, Holler-Kriz]

$\underline{\pi}_\blacklozenge H_K \underline{\mathbb{F}}_2$ is



$\underline{\text{Kernel}}$ is

$$\begin{array}{ccccccc} & \mathbb{Z}/4 & & & & \mathbb{F}_2\{b_L, b_R\} & \\ \mathbb{Z}/2 & \mathbb{Z}/2 & \mathbb{Z}/2 & \oplus & 0 & 0 & 0 \\ & 0 & & & & 0 & \end{array}$$

Compute $\underline{\pi}_\blacklozenge H_K \underline{\text{kernel}}$ & use LES.

Other related work

[Keyes] $\underline{\pi}_\blacklozenge H_K \underline{A} = \underline{\pi}_\blacklozenge H N_e^K \underline{\mathbb{Z}}$

$\downarrow$

[Slone] $\underline{\pi}_\blacklozenge H_K \underline{\mathbb{Z}}$

In progress:

[G-Keyes-Mehrle]

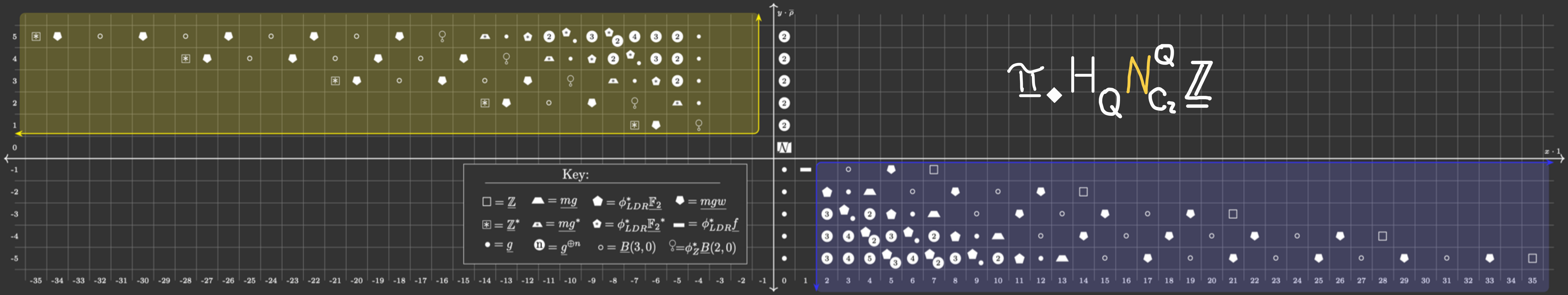
$$\underline{\pi}_0 N_{C_2}^{Q_8} MU_{\mathbb{R}} \cong N_{C_2}^{Q_8} \underline{\mathbb{Z}} = \begin{array}{ccc} & \mathbb{Z} \oplus \mathbb{F}_2\{b_L, b_R\} & \\ \mathbb{Z} & \mathbb{Z} & \mathbb{Z} \\ & \mathbb{Z} & \\ & \mathbb{Z} & \end{array}$$

$b_L - 2$, $b_L + b_R + 2$, $b_R - 2$, 2, 2, 2, 2

$\underline{\pi}_\blacklozenge H_Q N_{C_2}^{Q_8} \underline{\mathbb{Z}}$

$\downarrow$

[G-Slone] $\underline{\pi}_\blacklozenge H_Q \underline{\mathbb{Z}}$


Key:
$\square = \underline{\mathbb{Z}}$
$= \underline{mg}$
$= \phi^*_{LDR}\mathbb{F}_2$
$= \underline{mgw}$
$= \underline{\mathbb{Z}}^*$
$= \underline{mg}^*$
$= \phi^*_{LDR}\mathbb{F}_2{}^*$
$= \phi^*_{LDR}\underline{f}$
$\bullet = \underline{g}$
$= \underline{g}^{\oplus n}$
$\circ = \underline{B}(3,0)$
$= \phi^*_{\mathbb{Z}}\underline{B}(2,0)$

Thanks!